Application number: ______________ Name: ______________

Entrance examination – Question sheet (Physics)
Space and Astronautical Science Program,
Graduate Institute for Advanced Studies, SOKENDAI

Question 1.

As shown in Figure 1-1, there is a uniform rod with the mass of M and the length of L, which leans against a vertical wall on a horizontal floor. The points A and B are the places where the rod contacting the wall and floor surfaces, respectively and G is the center of gravity. When the rod is tilted, at what angle θ to the horizontal will the rod start to slide? Answer the following questions.

Let g be the gravitational acceleration, μ_1 and μ_2 be the coefficients of static friction between the rod and wall or floor, and N_1 and N_2 be the normal forces on the rod at the points A and B. (The subscripts 1 and 2 mean the wall and floor surfaces, respectively. The same shall apply hereinafter.)

Write the answer as well as all rationales and processes of derivations.

(1)-i

Express the condition where the rod starts to slide using F_1 and F_2, the frictional forces on the rod at points A and B, respectively. Assume that F_1 and F_2 reach the maximum at the same time.

(1)-ii

Show the equations of balanced forces and moments applied on the rod and find θ where the rod starts to slide down. The same conditions as in (i) apply.

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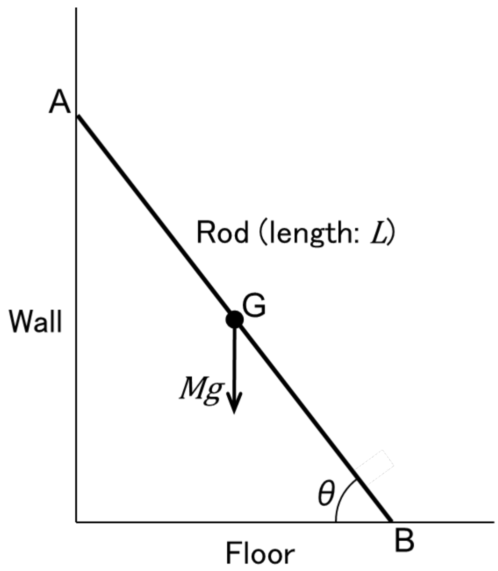

Figure 1-1 Uniform rod with the mass of M and the length of L, which leans against a vertical wall on a horizontal floor.

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Consider the oscillation of a rigid body (physical pendulum) moving around a horizontal fixed axis by the action of gravity as shown in Figure 1-2. Assume that the friction between the fixed axis and rigid body is negligible. Answer the following questions.

Let M be the mass of the rigid body, I be the moment of inertia around the horizontal fixed axis O (vertical to the paper), L be the distance between O and the center of gravity G, and g be the gravitational acceleration. Write the answer as well as all rationales and processes of derivations.

For the following questions, use the power series expansions of trigonometric functions below:

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots, \qquad \cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$$

(2)-i

Express the equation of motion of the physical pendulum using the oscillation angle θ and period t.

(2)-ii

When the oscillation angle θ of the physical pendulum is very small, find the solution of the oscillation angle by simplifying the equation of motion in (2)-i. In addition, find the natural angular frequency ω_0 and initial phase φ. Let θ_0 be the maximum angle of the pendulum and θ_1 be the initial angle at $t = 0$.

(2)-iii

Once again, find the solution of the oscillation angle θ when it is very small, but this time based on the law of conservation of energy. (The total energy E is the sum of potential energy U and kinetic energy K.)

As in question (2)-ii, let θ_0 the maximum angle of the pendulum and θ_1 be the initial angle at $t = 0$. The same processes as (2)-ii can be omitted by stating "same as (2)-ii".

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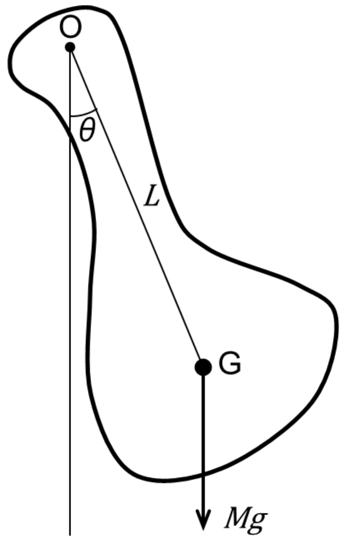

Figure 1-2 Physical pendulum with the mass of M, where I is the moment of inertia and L is the distance between the fixed axis O and the center of gravity G

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Question 2.

Magnets have the N and S poles, however, sometimes we can understand things easier by assuming that there exist point magnetic charges $\pm m$ on the poles ($+m$ on the N pole and $-m$ on the S pole, where $m > 0$). Then, the force $\boldsymbol{F}$ applied to the point magnetic charge m in a magnetic field $\boldsymbol{H}$ is:

$$\boldsymbol{F} = m\boldsymbol{H}$$

In vacuum, the magnetic field $\boldsymbol{H}$ produced by the point magnetic charge m at a point P separated by a distance r is:

$$\boldsymbol{H} = \frac{1}{4\pi\mu_0}\frac{m}{r^2}\boldsymbol{e}$$

where μ_0 is the magnetic permeability in vacuum, and $\boldsymbol{e}$ is the unit vector from the point magnetic charge to the point P. The magnetic dipole moment M of the magnet is:

$$M = ml$$

where l is the distance between the point magnetic charges $\pm m$ on the N and S poles. Using the above, answer the following questions.

(1)

As shown in Figure 2-1, suspend the center of the bar magnet horizontally by a string, and point the N pole of the magnet to the geomagnetic north. Then, rotate the bar magnet by a very small angle on the horizontal plane and release it slowly. The bar magnet will execute simple harmonic motion (oscillation) on the horizontal plane around the geomagnetic north-south direction. Find the period T of this simple harmonic motion, where M is the magnetic dipole moment of the bar magnet, I is the moment of inertia of the bar magnet for rotation around the string, and H_{E} is the geomagnetic field strength on the horizontal plane. Assume that the vertical component of the geomagnetic field is negligible, the rigidity of the string is negligible, and the bar magnet can rotate freely on the horizontal plane.

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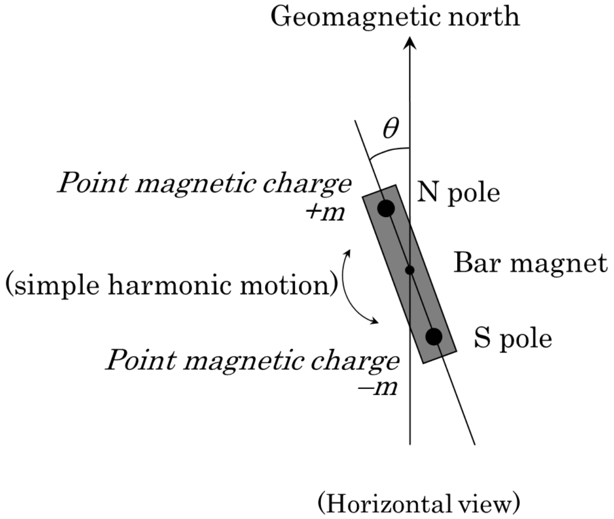

Figure 2-1 Bar magnet suspended by a vertical string

(2)

Place the bar magnet used in (1) on a horizontal plane in the atmosphere as shown in Figure 2-2. Answer the following questions regarding a point P, which is separated by a distance r from the center of the bar magnet on the line connecting the N and S poles.

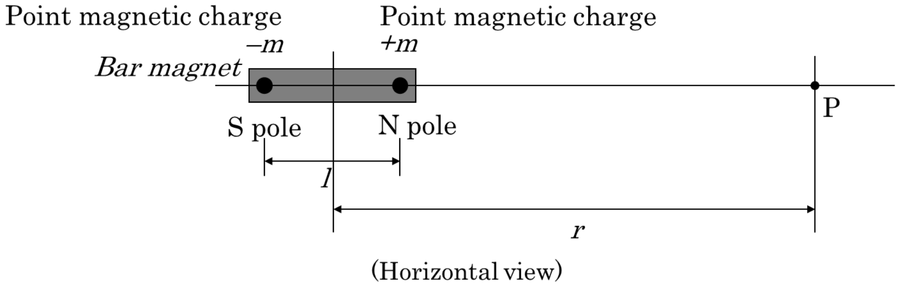

Figure 2-2 Bar magnet on the horizontal plane

(2)-i

Which is the direction of the magnetic field produced by the bar magnet at the point P? Indicate the direction at the point P with an arrow on the figure of the answer sheet with the reasons.

(2)-ii

Express the magnetic field strength H produced by the bar magnet at the point P by using M, the magnetic dipole moment of the bar magnet, r and

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μ_0. Assume that the magnetic permeability in the atmosphere is equal to that in vacuum μ_0, and r is sufficiently larger than the distance between point magnetic charges l.

To find the result, consider up to the first-order Taylor polynomial for (l/r), using the following Taylor expansion for the function $f(x)$ when $x = 0$:

$$f(x) = f(0) + \frac{1}{1!}f^{(1)}(0)x + \frac{1}{2!}f^{(2)}(0)x^2 + \frac{1}{3!}f^{(3)}(0)x^3 + \cdots$$

where $f^{(n)}$ in the Taylor expansion is the n-order derivative.

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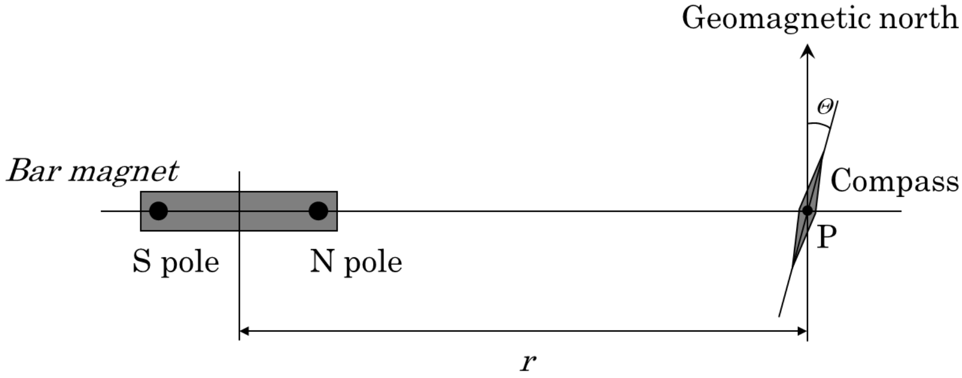
(3)

Place the bar magnet in Figure 2-2 so that the line connecting the N and S poles of the bar magnet is perpendicular to the geomagnetic north-south direction. Then, place a compass at a point P on the horizontal plane. As shown in Figure 2-3, the N pole of the compass points at an angle Θ east of the geomagnetic north. Answer the following questions. Consider only the component of the magnetic field on the horizontal plane and assume that the magnetic field produced by the compass is negligible.



(Horizontal view)

Figure 2-3 Bar magnet and compass at the point P

(3)-i

Express the angle Θ using H_E, the geomagnetic field strength at the point P, and H, the magnetic field strength produced by the bar magnet.

(3)-ii

Using the results of (1), (2), and (3)-i, express the geomagnetic field strength H_E with I, T, Θ and r, where I is the moment of inertia of the bar magnet, T is the period of simple harmonic motion in the geomagnetic field, Θ is the angle of compass needle, and r is the distance between the bar magnet and compass.

In addition, express the magnetic dipole moment M of the bar magnet with I, T, Θ, and r.

(This material is collected after the exam.)