Application number: Name:

Entrance examination – Question sheet (Mathematics)
Space and Astronautical Science Program, Graduate Institute for Advanced Studies, SOKENDAI

Question 1-1

(1) Answer the following questions. Write the answer as well as the process of derivation.

i. Find the matrix X that satisfies $AX=B$ using the cofactor method, where the matrices A and B are:

$$A = \begin{pmatrix} 0 & 1 & 2 \\ -3 & 1 & -1 \\ 1 & 2 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 3 & -2 \\ -6 & -8 \\ 3 & 1 \end{pmatrix}$$

ii. Find the matrix X that satisfies $AX+B=O$ using raw reduction, where the matrices A and B are:

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

iii. Find the eigenvalue and the characteristic vector, where the matrix A is:

$$A = \begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 0 \\ 1 & 3 & -2 \end{pmatrix}$$

iv. Find the solution of the following equation.

$$\begin{pmatrix} 2 & 3 & 1 \\ 4 & 1 & -3 \\ -1 & 2 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix}$$

Use that $A=BC$ holds when the matrices A, B, and C are as follows:

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 4 & 1 & -3 \\ -1 & 2 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 0 & 0 \\ 4 & -5 & 0 \\ -1 & 3.5 & -1 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 1.5 & 0.5 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix},$$

v. Express x_1 *and* x_2 using z_1 and z_2 based on the concept of matrices when the following relations exist:

$$x_1 = a_{11}y_1 + a_{12}y_2 + a_{13}y_3$$
$$x_2 = a_{21}y_1 + a_{22}y_2 + a_{23}y_3$$

$$y_1 = b_{11}z_1 + b_{12}z_2$$
$$y_2 = b_{21}z_1 + b_{22}z_2$$
$$y_3 = b_{31}z_1 + b_{32}z_2$$

(* This material is collected after the exam.)

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(2) Find the following determinants. Write the answer as well as the process of derivation.

i.

$$\begin{vmatrix} a & 0 & c \\ b & c & 0 \\ 0 & a & b \end{vmatrix} =$$

ii.

$$\begin{vmatrix} 8 & 0 & 0 & 0 & 0 \\ 10 & 1 & 0 & 0 & 0 \\ 5 & 9 & 3 & 0 & 0 \\ 2 & 2 & 8 & 7 & 0 \\ 9 & 5 & 13 & 11 & 20 \end{vmatrix} =$$

iii.

$$\begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix} =$$

iv.

$$\begin{vmatrix} a & b & b & b & b \\ b & a & b & b & b \\ b & b & a & b & b \\ b & b & b & a & b \\ b & b & b & b & a \end{vmatrix} =$$

(3) Find λ that allows solutions other than $a_1 = a_2 = a_3 = a_4 = 0$ in the following simultaneous equations. Write the answer as well as the process of derivation.

$$-a_1 + a_2 = 0$$
$$-a_1 + a_2\lambda + a_3 = 0$$
$$a_2 - a_3\lambda + a_4 = 0$$
$$a_3 - a_4\lambda = 0$$

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Question 2-1. Find the derivative $\frac{df}{dx}$ of the function $f(x)$. Write the answer as well as the process of derivation.

(1) $f(x) = \sqrt{x^2 - 4x + 5}$

(2) $f(x) = \dfrac{\cos x}{1+\tan x}$

(3) $f(x) = \dfrac{4x+1}{(x-1)^2}$

(4) $f(x) = (x+3)^x$

Question 2-2. Find the indefinite integral $\int f(x)\,dx$ of the function $f(x)$, where log is the natural logarithm and C is a constant of integration. Write the answer as well as the process of derivation.

(1) $f(x) = \dfrac{3x^5}{(1+x^3)^3}$

(2) $f(x) = 3x^2(\log x)^2$

(3) $f(x) = \dfrac{e^{-x}}{e^{-x}+1+e^x}$

Question 2-3. Find the limit $\lim_{n\to\infty} a_n$.

(1) $a_n = \dfrac{1}{n^2}\ (1 + 2 + \cdots + n)$

(2) $a_n = 1 + \frac{3}{8} + \frac{1}{5} + \cdots + \dfrac{3}{(n+1)^2 - 1}$

(3) $a_n = \displaystyle\int_{2n}^{3n} \frac{x+3}{x^2+1}\,dx$

Question 2-4. There is a triangle circumscribed by a circle of radius *r*.

(1) Express the area of the triangle as a function of $r, \theta, and\ \varphi$, where $\varphi\ and\ \Theta$ represent angles of two vertices of the triangle.

(2) Describe the features of the triangle when the area is the minimum for a given r.

(* This material is collected after the exam.)