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Entrance examination – Question sheet (Physics)
Department of Space and Astronautical Science,
School of Physical Sciences, SOKENDAI

Question 1 Answer the following questions. Objects are point masses and the only force applied on the objects is gravity. Let $\sqrt{2}$ be 1.41 and $\sqrt{3}$ be 1.73 for calculation as needed.

(1)-A Assuming that the ground is an infinite horizontal plane, an object with a mass of m is ejected from a height of h with a velocity of V in the horizontal direction. Calculate the horizontal traveling distance of the object when it reaches the ground (See Figure 1-1) where the gravity is always directed vertically downward to the ground and gravitational acceleration is g.

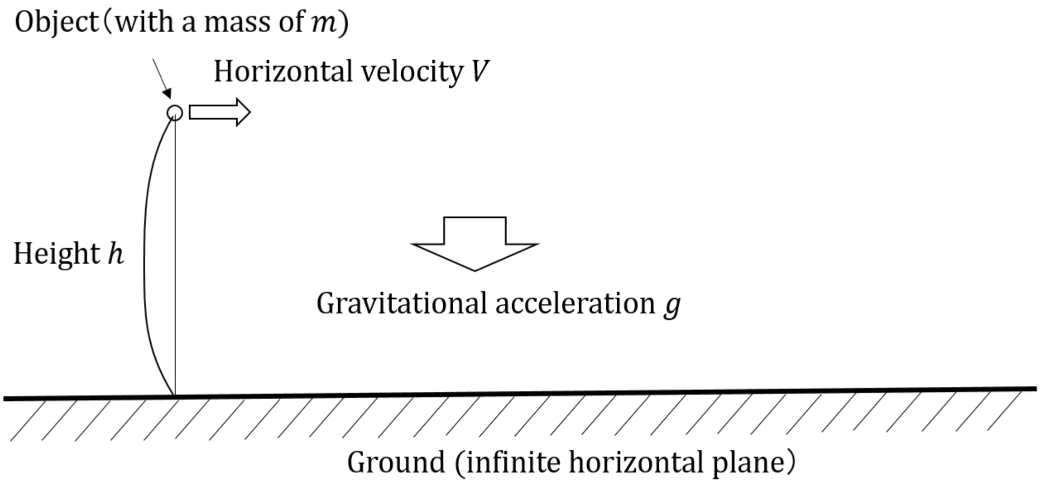

Figure 1-1 Motion of an object with a mass of m ejected in the horizontal direction

(1)-B Reduce the gravity force to 1/6 of that in Question 1-A. Calculate how many times the velocity required to reach the same location is higher than that in Question 1-A to two significant figures.

In the following questions (2), (3), and (4), consider motion of an object traveling around a celestial body with a mass of M and a radius of R as shown in Figure 1-2. Assume that gravitational constant is G and the mass of the object m is sufficiently small compared to the mass of the celestial body M.

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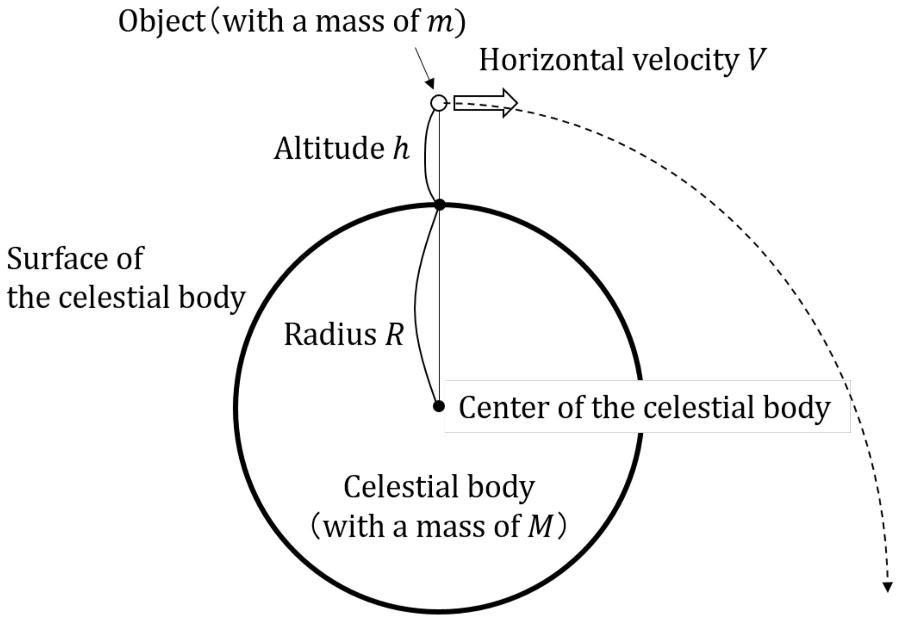

Figure 1-2 Motion of an object with a mass of m traveling around a celestial body

(2)-A Apply a velocity in the horizontal direction to the object at a distance of h away from the celestial body in the vertical upward direction (hereinafter referred to as altitude of h). Find the velocity V_c required for the object to travel at a constant altitude. Also, find the time T_c until the object returns to the position where the speed is initially applied.

(2)-B When applying a velocity in the horizontal direction to the object at the altitude of h, find the minimum velocity V_p required for the object to go infinitely away and not to return the original position. Also, calculate how many times the velocity is as high as V_c obtained in Question (2)-A to two significant figures.

(3) There is an object with a mass of m traveling at a velocity of V_c on a circular orbit at the altitude of h around the celestial body shown in Figure 1-2. By momentarily accelerating (or decelerating) the object in the traveling direction, the time until the object returns to the point of acceleration (or deceleration) becomes K (more than 1) times longer than T_c. Find how many times the velocity of the object required to achieve the above is higher than the velocity on the circular orbit, V_c .

You can use the following equation, which provides energy of an object with a mass of m traveling around a celestial body with a mass of M using the semi-major axis

a of the orbit.

$$E = -\frac{GmM}{2a}$$

You can also use the following equation, which expresses the relationship between the period of revolution T of the object and the semi-major axis a.

$$\frac{T^2}{a^3} = \frac{4\pi^2}{GM}$$

(4)-A There is an object with a mass of m travelling at a velocity of V_c on a circular orbit at the altitude of h around the celestial body shown in Figure 1-2. Consider momentary application of a velocity to the object to reach the surface of the celestial body. Express the minimum change of the velocity required to reach the surface of the celestial body using V_c , R, and h for the cases where a velocity toward the center of the celestial body is applied to the object and where the object is decelerated in the traveling direction, respectively.

(4)-B Show that regardless of the altitude of the circular orbit, the minimum change of the velocity required for the object to reach the surface of the celestial body is always smaller in the case where the object is decelerated in the traveling direction compared to the case where the object is accelerated toward the center of the celestial body.

(4)-C Assume that the speed V_c is $800\,m/s$ and the ratio of the altitude of the circular orbit to the diameter of the celestial body $h/_R$ is $1/_{12}$. Calculate the deceleration ΔV (m/s) in the traveling direction required for the object to reach the surface of the celestial body to two significant figures.

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Question 2 There is a magnetic field in the positive direction of the z-axis with slowly changing intensity as shown in Figure 2-1. Consider cyclotron motion of a charged particle with the center of gyration on the z-axis. Assume that the magnetic field intensity B_z and $\frac{\partial B_z}{\partial z}$ are constant in the space scale of cyclotron motion of charged particles.

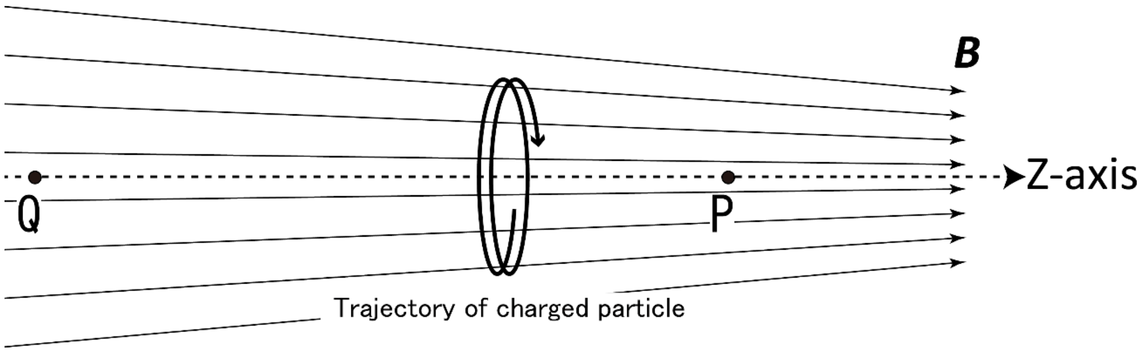

Figure 2-1 Cyclotron motion of a charged particle in a magnetic field

(1) Let charge of the charged particle be q, mass be m, and velocity in the direction vertical to the magnetic field be v_ϕ. Show that Larmor radius (radius of gyration) of the charged particle R_L can be expressed as follows:

$$R_L = \frac{mv_\phi}{qB_z}$$

It is noted that Lorentz force F is applied to a charged particle moving in a static magnetic field $\boldsymbol{B}$, which is expressed as follows:

$$F = q\boldsymbol{v} \times \boldsymbol{B}$$

where $\boldsymbol{v}$ is the velocity vector of the charged particle.

(2) The magnetic field has divergence equal to zero ($\nabla \cdot \boldsymbol{B} = 0$). Consider a cylinder having the axis along the z-axis with sufficiently small ΔZ as shown in Figure 2-2, and the integral of the magnetic field intensity in the normal vector across the overall surface. Show that the magnetic field intensity B_R vertical to the z-axis on the cylinder surface at the distance of R_L away from the z-axis is expressed as follows:

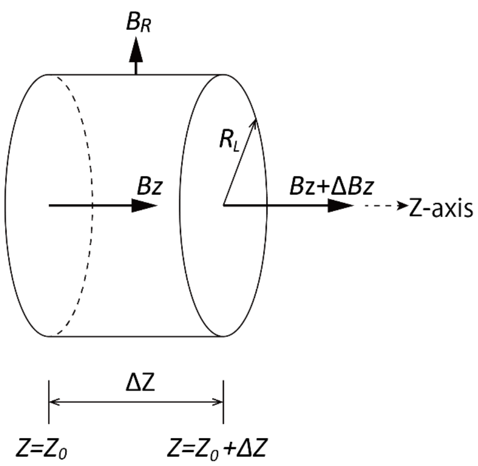

Figure 2-2 Magnetic field on the cylinder surface

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$$B_R = -\frac{1}{2} R_L \frac{\Delta B_z}{\Delta Z}$$

In Figure 2-2, B_z is the magnetic field intensity in the z-axis direction when $Z = Z_0$ and $B_z + \Delta B_z$ is that when $Z = Z_0 + \Delta Z$. Assume that B_R is constant on the cylinder surface and B_z changes linearly to Z.

(3) On the gyration orbit of the charged particle, there is B_R unequal to zero. Therefore, Lorentz force F_z in the direction of the magnetic field line is applied to the charged particle. Show that the force F_z is expressed by the following equation:

$$F_z = -\frac{mv_\phi^2}{2B_z} \frac{\Delta B_z}{\Delta Z}$$

(4) No work is done on the charged particle by the magnetic field, therefore, the law of conservation of energy can be applied as follows:

$$\frac{d}{dt}\left(\frac{1}{2} m v_\phi^2\right) + \frac{d}{dt}\left(\frac{1}{2} m v_z^2\right) = \frac{d}{dt}\left(\frac{1}{2} m v_\phi^2\right) + m \frac{dz}{dt} \frac{dv_z}{dt} = 0$$

where t is time, and v_z is the velocity of the charged particle in the z-axis direction. Thus, acceleration/deceleration in the z -axis direction decelerates/accelerates the speed of gyration. Based on the above, prove the following equation.

$$\frac{mv_\phi^2}{2B_z} = \text{const}$$

(5) The angle between the velocity vector of the charged particle having the center of gyration at a point P (see Figure 2-1) on the z-axis and the direction of the magnetic field is 90 degrees now. The charged particle, while in cyclotron motion, travels to the direction where the magnetic field weakens due to the force F_z in the direction of the magnetic field lines. Find the angle α between the velocity vector of the charged particle and the direction of the magnetic field when the center of gyration of the charged particle reaches the point Q. Let the magnetic field intensities at the points P and Q be B_P and B_Q, respectively.