Application number: Name:

Entrance examination – Question sheet (Physics)
Department of Space and Astronautical Science, School of Physical Sciences, SOKENDAI

Question 1

Consider the vertical motion of an airplane with two wings (a main wing and a tail wing). When the air density is ρ, the velocity of the air flow from the front is v, and the angle between the air flow and each wing is θ, the main and tail wings generate upward forces of $\frac{1}{2}\rho v^2 C_W \theta$ and $\frac{1}{2}\rho v^2 C_T \theta$ perpendicular to the air flow, respectively. The working points of the forces generated by these wings and the center of gravity of the airplane are on the baseline of the airplane, and the distance from the nose of the airplane to the working point of the force from the main wing, the center of gravity, and the working point of the force from the tail wing is L_W, L_G, and L_T, respectively (see the following figure). The main wing is fixed at an angle of 0 (zero) to the baseline whereas the tail wing can be tilted to a desired angle. The mass of the airplane is m. Assume that the air drag, the aerodynamic force generated by parts other than the wings such as the fuselage(*) of the airplane, and effects of winds other than the velocity of the airplane itself are negligible, and that the aerodynamic forces generated by the main wing and the tail wing are independent, do not interfere with each other, and have no effect on the air flow.

(*) fuselage : the main part of a plane, in which people sit or goods are carried

Question 1-1

Consider the state where the above airplane is flying horizontally while balancing the gravity with the forces generated by the main and tail wings (hereinafter referred to as steady level flight). Show the equations representing i) balance of the vertical forces and ii) balance of moment around the center of gravity, using α as the angle between the baseline of the airplane and the velocity (angle of attack), δ as the angle between the baseline of the airplane and the tail wing (tail wing angle), and v as the velocity. See the following figure for the directions of α and δ. Let the air density be ρ and the gravitational acceleration be g. These definitions are common in the following questions.

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Question 1-2

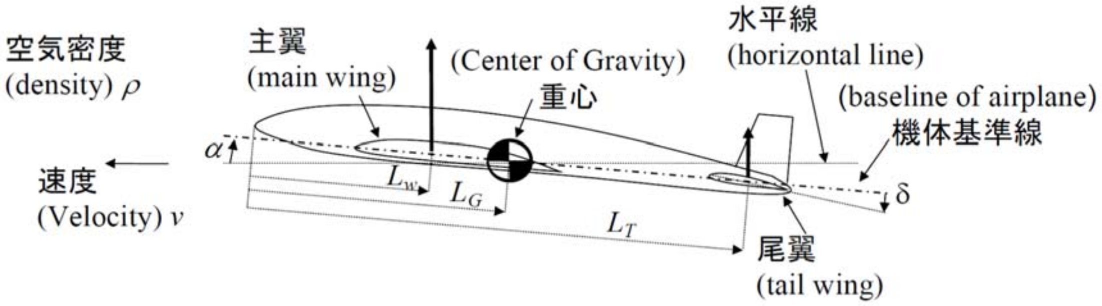
Find the tail wing angle δ_1 and the angle of attack α_1 for steady level flight when $v = v_1$.

Question 1-3

During steady level flight with the velocity of v, the angle of attack of α, and the tail wing angle of δ, the angle of attack changed to $\alpha+\Delta\alpha$ due to a blast, etc. Determine the moment ΔM around the center of gravity generated in this case (the direction is defined as the same as α) and show the necessary relationships among L_W, L_G, L_T, C_W, and C_T to prevent the angle of attack α from diverging. Assume that $\Delta\alpha$ is very small and you can use $\cos\Delta\alpha = 1, \sin\Delta\alpha = 0$ for simplifying.

Question 1-4

During steady level flight with the velocity of v, the angle of attack of α, and the tail wing angle of δ at the altitude of h_0, the velocity changed to $v_2 = v - \Delta v < v$ due to a blast, etc. In this case, a vibration in the vertical direction occurs as follows: the gravity overwhelm the forces generated by the wings and the airplane is going down with increasing the velocity, and at a point, the forces generated by the wings exceeds the gravity and the airplane is going up with decreasing the velocity. Then, when the forces generated by the wings went below the gravity again, the airplane starts to go down. We can say that this vertical vibration is caused by exchanging between the potential energy and the kinematic energy. Determine the velocity v_h at the attitude h and show the relationship between the period of this vibration and the velocity v of steady level flight. Assume that the vertical velocity of this vibration is very small compared to the velocity v and v_2, the change in the direction of the airflow due to the vertical velocity is negligible, and that the air density ρ and the gravitational acceleration g are constant regardless of the altitude.



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Question 2-1

When magnetic flux through a closed-circuit changes with time, an electromotive force is generated in the circuit. The generated electromotive force is called the induced electromotive force, and it is expressed based on Faraday's law as follows:

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t}$$

where $\mathcal{E}$ is the induced electromotive force, Φ is magnetic flux, and t is time.

Show that one of Maxwell equations

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

is equivalent to Faraday's law, where $\boldsymbol{E}$ is an electric field, and $\boldsymbol{B}$ is a magnetic flux density.

Consider only the case that the magnetic flux density $\boldsymbol{B}$ thorough the closed-circuit changes with time. You do not need to consider movement or deformation of the circuit.

And you can use Stoke's theorem:

$$\oint_C \boldsymbol{A} \cdot d\boldsymbol{l} = \iint_S \nabla \times \boldsymbol{A} \cdot d\boldsymbol{S}$$

where

$$\oint_C \boldsymbol{A} \cdot d\boldsymbol{l}$$

is a line integral of the vector $\boldsymbol{A}$ along the closed-circuit C, and

$$\iint_S \boldsymbol{B} \cdot d\boldsymbol{S}$$

is a surface integral of the vector $\boldsymbol{B}$ over the surface S surrounded by the closed-circuit C.

* In the following questions, you can use the following equations:

- The magnetic moment generated by a circular coil carrying the current of I can be expressed as $\boldsymbol{m} = IS\boldsymbol{n}$, where S is the area of the circular coil, and $\boldsymbol{n}$ is the normal unit vector on the coil plane.
- The sense of the vector $\boldsymbol{n}$ is given by the right-hand screw rule, i.e., it is in the direction in which a screw would move forward if rotating in the same direction as the current flow.
- The magnetic field exerts a torque on a closed-circuit carrying current. The torque $\boldsymbol{N}$ to the circuit in the uniform magnetic field $\boldsymbol{B}$ is expressed as $\boldsymbol{N} = \boldsymbol{m} \times \boldsymbol{B}$ using the magnetic moment $\boldsymbol{m}$ of the circuit.
- The counter electromotive force due to self-inductance is negligible.

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Question 2-2.

(1) Place a circular coil with a radius of a in the xy-plane. The center of the circular coil is set as the origin of the xy-plane, and a uniform static magnetic field $\boldsymbol{B}$ is directed toward the z-axis. Rotate the circular coil around the x-axis at the angular velocity of ω (See Figure 1). Determine the induced electromotive force $\mathcal{E}$ generated in the circular coil at t, where t is time. Also, determine the current I flowing in the coil using R as the electrical resistance of the coil.

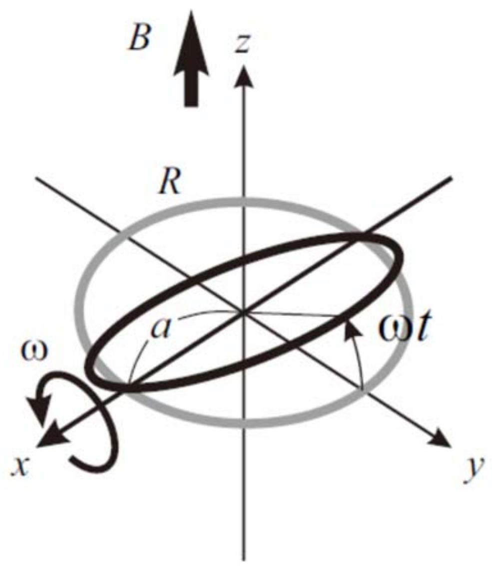

Figure 1

(2) Determine the magnetic moment $\boldsymbol{m}$ of the circular coil at t and the torque $\boldsymbol{N}$ exerted by the magnetic field $\boldsymbol{B}$ on the circular coil.

Question 2-3

Place a spherical electrically-conducting shell (its radius is a, and its thickness is b, where $a \gg b$) and the center of the spherical shell is set as the origin of the coordinates. A uniform charge is distributed with a density of ρ on the spherical shell. The spherical shell rotates around the x-axis at the angular velocity of ω. Determine the electric current density $J(\theta)$ at each θ, considering a circular current between the angles θ and $\theta + \delta\theta$, where θ is the angle from the x-axis as shown in Figure 2.
Also, determine the overall magnetic moment $\boldsymbol{m}$ of the rotating spherical shell by integrating the partial magnetic moment $d\boldsymbol{m}$ generated by each circular current with respect to θ.

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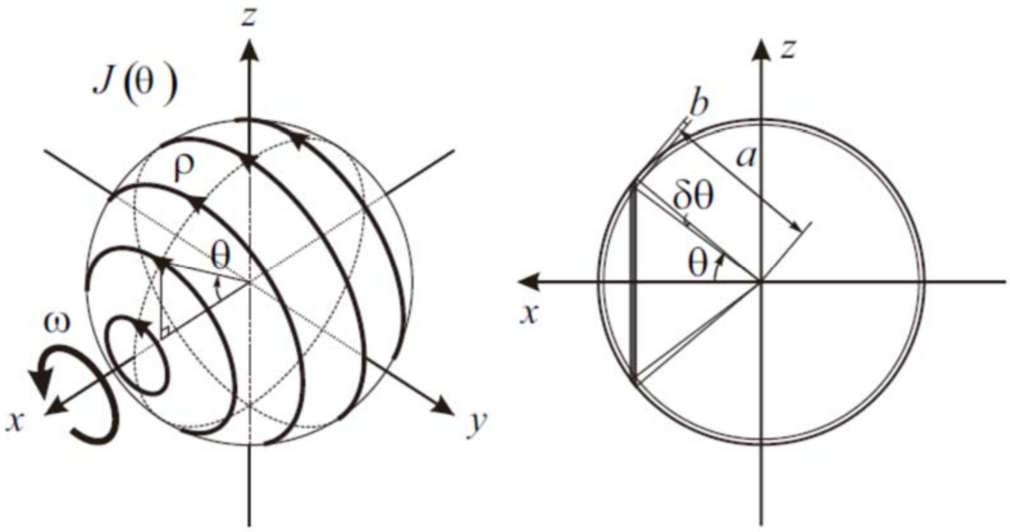

Figure 2

Question 2-4
When moving an electrical conductor in a magnetic field, eddy currents are induced in the electrical conductor by electromagnetic induction. Consider eddy currents generated in a spherical electrically-conducting shell that rotates in a uniform magnetic field.

(1) Place a spherical shell (its radius is a, and its thickness is b, where $a \gg b$) in a uniform static magnetic field $\boldsymbol{B}$ directing the z-axis and the center of the spherical shell is set as the origin of the coordinates. Rotate the spherical shell around the y-axis at the angular velocity of ω. In this case, induced currents caused by the induced electromotive force generated in the spherical shell flow on the circumferences which are the intersections between the spherical shell and planes parallel to the yz-plane as shown in Figure 3. Explain the reason qualitatively.

(2) Determine the induced electromotive force $\mathcal{E}(\theta)$ generated at each θ, considering a circular coil between the angles θ and $\theta + \delta\theta$, by calculating the change in the magnetic flux through the circular coil with the rotation of the spherical shell, where θ is the angle to the x-axis as shown in Figure 3. Also, determine the current density $J(\theta)$ at θ using σas the electrical conductivity of the spherical shell.

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(3) Using $J(\theta)$ in (2), determine the overall magnetic moment $\boldsymbol{m}$ of the rotating spherical shell by integrating the partial magnetic moment $d\boldsymbol{m}$ generated by each circular current with respect to θ. Also, determine the torque $\boldsymbol{N}$ exerted by the magnetic field $\boldsymbol{B}$ to the rotating spherical shell.

(4) Explain how the angular velocity changes with time due to the torque determined in (3). Here, the rotational inertia of the spherical shell around the rotation axis is expressed as M.

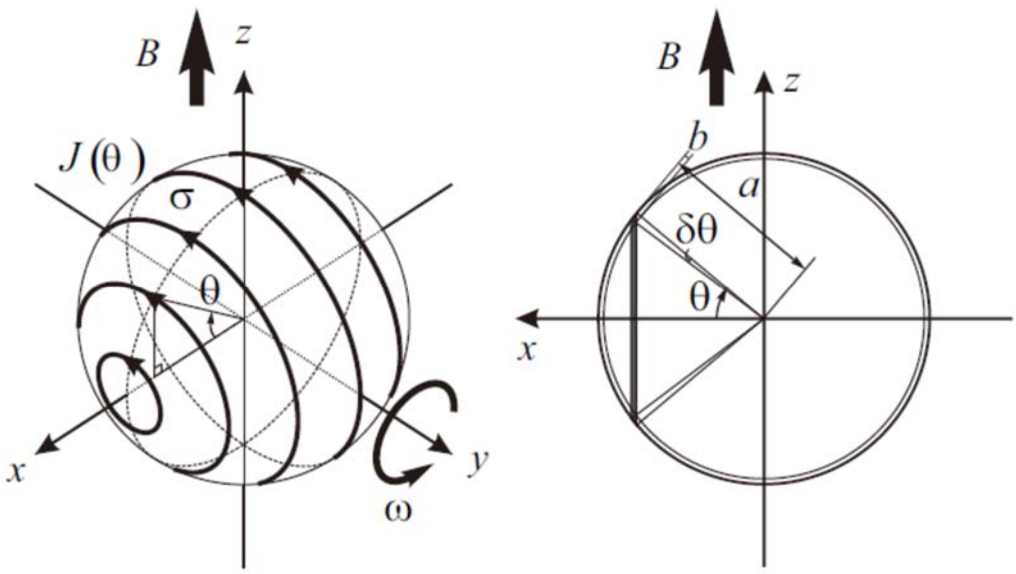

Figure 3