Application number: Name:

Entrance examination – Question sheet (Mathematics)
Department of Space and Astronautical Science,
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Question 1-1.

Answer the following questions. Write the answer as well as the process of derivation.

(1) Differentiate the following function.

$$y = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

(2) Find the partial derivatives of the following function with respect to each of x and y.

$$z = \log_x \sqrt{y^2+1}$$

(3) Calculate the following definite integral substituting x by $\sinh(t)$, where $\sinh(t) = \frac{1}{2}(e^t - e^{-t})$.

$$\int_0^1 \sqrt{1+x^2}\, dx$$

Question 1-2.

(1) When $y = f(x)$ is differentiable in the period of $[\alpha, \beta]$ and $f'(x)$ is a continuous function, show that the area of the curved surface S obtained by revolution of this curve around the x-axis is as follows, where $f(x) \geq 0$.

$$S = 2\pi \int_\alpha^\beta f(x)\sqrt{1+\{f'(x)\}^2}dx$$

(2) A cycloid is the curve traced by a point on the rim of a circle when the circle rolls along a straight line without slipping. The parametric representation is $x =$

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$r(\theta - \sin\theta)$ and $y = r(1 - \cos\theta)$, where the radius of the circle is r and the rotation angle is θ.

Calculate the area of the curved surface obtained by revolution of this curve in the range of $0 \leq \theta \leq 2\pi$ around the x-axis.

Question 1-3.

Calculate the following double integral converting it to polar coordinates.

$$\iint_D e^{-x^2-y^2}\,dxdy$$

$$D: x^2+y^2 \leq 1, x \geq 0, y \geq 0$$

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Question2-1. Show that eigenvalues of the matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ are 1, ζ, and ζ^2, where ζ is a solution of $x^2 + x + 1 = 0$.

Qusestion2-2. Express the circulant matrix $B = \begin{pmatrix} a_1 & a_2 & a_3 \\ a_3 & a_1 & a_2 \\ a_2 & a_3 & a_1 \end{pmatrix}$ using a unit matrix E as well as A and A^2 in Question 2-1.

Question 2-3. Assume that the matrix A in Question 2-1 can be diagonalized by a matrix P as follows: $P^{-1}AP = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \zeta & 0 \\ 0 & 0 & \zeta^2 \end{pmatrix}$

where ζ is defined as Question 2-1 and P^{-1} is the inverse matrix of P.

Express the eigenvalues of the matrix B in Question 2-2 using a_1, a_2, a_3, ζ, and ζ^2.

Question 2-4. Let the determinant of the matrix B in Question 2-2 be $|B|$. When $a_1 + a_2 + a_3 \neq 0$ and $\sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2} = 1$, find all real numbers $a_1,\ a_2, and\ a_3$ satisfying $|B| = 0$.

Question 2-5. Specifically calculate the matrix P in Question 2-3, where P is a unitary matrix defined as $P^{\dagger}P = E$. The symbol $\dagger$ on the upper right side represents the complex conjugate transpose of the matrix P.

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