## Entrance examination - Question sheet (Physics) <br> Department of Space and Astronautical Science, School of Physical Sciences, SOKENDAI

## Question 1

(1) As shown in Figure 1, there is a uniform rod with the mass of $M$, the length of $L$, which is suspended horizontally from the ceiling by 4 strings of the same length, thickness, and material at 4 points: the both ends of the rod and the points $L / 3$ away from both ends. Assume that the ceiling is also horizontal, the mass of strings is negligible, and the strings stretch in proportion to tension although the amount is infinitesimal compared to $L$. In this situation, tension applied to one string is $M g / 4$, where $g$ is the gravitational acceleration.

When the leftmost string is cut, the rod is suspended by remaining 3 strings. Calculate tension applied to each of the strings.


Figure 1
(2) There is a spherical celestial body A with the radius of $R$ and the mass of $M$. As shown in Figure 2, consider movement of a celestial body $B$ of which relative velocity at infinity is $v_{\infty}$ with respect to the celestial body A and the mass is negligible compared to the celestial body A. At infinity, the celestial body B moves in the horizontal direction shown in the Figure, and the distance from the straight line which passes through the center of the celestial body A and is parallel to the movement direction of the celestial body B (horizontal dash-dotted line shown in the Figure) is $\sigma$.


Figure 2

Express the condition of $\sigma$ where the celestial body B collides against the celestial body A using $R, M, v_{\infty}$ and the universal gravitational constant $G$. Assume that only the gravity of the celestial body A influences movement of the celestial body B.
(3) Instead of the celestial body $B$ in the above question (2), there is a group of celestial bodies at infinity which is comprised from infinitesimal celestial bodies and of which the mass density is $\rho$ and the relative velocity with respect to the celestial body A is $v_{\infty}$ (Figure 3). Calculate the mass of the group of celestial bodies which collides against the celestial body A per unit time. Assume that only the gravity of the celestial body $A$ influences movement of the group of celestial bodies.


Figure 3

## Question 2-1

(1) As shown in Figure 1, dielectric materials with the dielectric constant of $\varepsilon$ are filled between coaxial cylindrical conductors with the radius of $a$ and $b$, which are sufficiently long. When these materials are charged by potential difference $V$, explain that the electric field distribution $E$ in the dielectric materials is given as the following formula, which is the function of the distance from the central axis $r$, using Gauss's law. Applying charge to the surface of the inner cylinder may help you to derive the formula.

$$
E=\frac{V}{r \log (b / a)}
$$



Figure 1

Also, calculate the electrostatic capacity per unit length.
(2) As shown in Figure 2, when there is a uniform electric field parallel to the boundary surface of two dielectric materials with dielectric constants of $\varepsilon_{1}$ and $\varepsilon_{2}$, the perpendicular force applied to the unit area of the boundary surface $f_{n}$ is expressed as follows:

$$
f_{n}=\frac{1}{2}\left(\varepsilon_{1}-\varepsilon_{2}\right) E^{2}
$$



Figure 2


Figure 3

## Question 2-2

(1) As shown in Figure 4, a uniform electric field directed upward and parallel to the plane of paper, $E$, exists in the region $S$. There is no electric field outside $S$. The cross section of the region $S$ on the plane of paper is a rectangle of which vertices include 3 points, $A, B$, and $D$ and the width of the region $S$ (distance between A and D ) is $L$. A charged particle which has the charge of $+q$ and the mass of $m$ travels horizontally with the velocity of $v$ and enters the region from the


Figure 4 left. After the particle goes out of $S$, it reaches the point $C$ which is located on the plate $L^{\prime}$ away from the side BD . In this situation, derive the distance $x$ between the point $C$ and the point $C^{\prime}$, which is the location where the particle reaches if it travels in the original, straight and horizontal direction. Assume that the particle passes below the point $B$ and the gravity is negligible.
(2) Consider the situation where a uniform magnetic field vertical to the plane of paper, $B$, exists in the region S described in (1) (Figure 5). There is no magnetic field outside S. When a charged particle which has the charge of $+q$ and the mass of $m$ travels horizontally with the velocity of $v$ and enters the region from the left, the magnetic field causes circular motion of the particle


Figure 5 with the radius of $r$ in the region $S$. When the particle moves as shown in Figure 5, the point $C$, which is the location where the particle reaches on the plate $L^{\prime}$ away from the side BD , is located $y$ above the point $\mathrm{C}^{\prime}$, which is the location where the particle reaches if it travels in the original, straight and horizontal direction. The distance $y$ is expressed by the following formula:

$$
y=h+L^{\prime} \tan \theta=\frac{L^{2} q B}{2 m v}+L^{\prime} L /\left(\frac{m v}{q B}-\frac{L^{2} q B}{2 m v}\right)
$$

where the particle passes below the point B . The radius of circular motion of the particle, $r$, is sufficiently large compared to the distance $h$ between the point where the particle goes out of the region $S$ and the point where the particle passes on the side BD if it travels in a straight direction. Assume that the gravity is negligible.

Using the above formula, answer the following question. In Figure 6, when uniform electric and magnetic fields directed upward and parallel to the plane of paper exist, explain that the points where the particles travelling at various velocities reach on a plate vertical to the particle incident direction form a parabola defined by $q / m$. Assume that $m v \gg q B$.


Figure 6

