A memo on the magnetic field density expressed in different units

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Magnetic energy density is written as

$$\epsilon\,[\mathrm{erg/cm^3}] = \frac{1}{8\pi}\,(B\,[\mathrm{gauss}])^2 \quad \text{(Gauss unit)} \tag{1}$$

or

$$\epsilon'\,[\mathrm{J/m^3}] = \frac{1}{2\mu_0}\,(B'\,[\mathrm{T}])^2 \quad \text{(MKSA unit)}, \tag{2}$$

where $\mu_0 = 4\pi \times 10^{-7}$ [N/A^2] is the magnetic permeability in vacuum. Here, in order to clarify difference of the same physical quantities expressed by two different units, we put $'$ for ϵ and B in the second equation.

Note that [gauss] has the dimension of [cm$^{-1/2}$ g$^{1/2}$ s^{-1}], so that [gauss^2] corresponds to [erg/cm^3]. Also, [T] has the dimension of [N/(A·m)], so that [T^2]/μ_0 corresponds to [J/m^3]. Magnetic flux density strength of 1 [T] is equal to 10^4 [gauss], thus $B' = 10^{-4}\,B$. However, **be careful that dimensions of [T] and [gauss] are different**:

$$1\ \mathrm{T}\ \text{(MKSA unit)} \Longleftrightarrow 10,000\ \text{gauss (cgs unit)}.$$

In the above equation, often the left-hand side and the righ-hand side are connected by equal sign, which, I belive, is mis-leading and should be avoided.

Note,

$$[\mathrm{J/m^3}] = [10^7 \mathrm{erg}/(100\mathrm{cm})^3] = 10\ [\mathrm{erg/cm^3}],$$

thus,

$$\epsilon' = 1/10\ \epsilon.$$

Similarly,

$$\epsilon' = \frac{1}{2\mu_0} B'^2 = \frac{1}{2\times 4\pi \times 10^{-7}}(10^{-4}\,B)^2$$
$$= \frac{1}{10}\frac{B^2}{8\pi} = \frac{1}{10}\epsilon.$$

Now, we see that equations (1) and (2) agree.