

1. Black body radiation

1. From what kinds of sources will we observe black body emission in X-ray?

熱の存在放射  
光学的に厚い天体 → 黒体放射

$$B_\nu(T) = \frac{2}{c^2} \frac{h\nu^3}{e^{h\nu/kT} - 1} \text{ [ergs/s/cm}^2\text{/Hz/str]}$$

中性恒星の表面, 白色矮星の表面, 活動的星

2. Derive the formula of the Planck function for the low-energy limit (Rayleigh-Jeans law) and high-energy limit (Wien function)

低エネルギー限界 (Rayleigh-Jeans law):  $h\nu \ll kT, B_\nu(T) \approx \frac{2}{c^2} \frac{h\nu^3}{1 + \frac{h\nu}{kT}} \approx \frac{2}{c^2} \frac{h\nu^3}{kT}$

高エネルギー限界 (Wien function):  $h\nu \gg kT, B_\nu(T) \approx \frac{2}{c^2} \frac{h\nu^3}{e^{h\nu/kT}}$

天体の中心部や中性恒星の中心部 光学的に厚い 降着円盤

3. Obtain the energy at which the black body energy spectrum (Planck function) peaks

Use the following formula:  $E_p \approx 3kT$  (おおよそ)  
https://www.wolframalpha.com/input/?i=%283+\*+x+%29+\*+exp%28-%29+\*3D+3&lang=en

$$\frac{\partial B_\nu}{\partial \nu} \propto \frac{3\nu^2(e^{h\nu/kT}-1) - \nu^3 e^{h\nu/kT} \cdot \frac{h}{kT}}{(e^{h\nu/kT}-1)^2} = 0$$

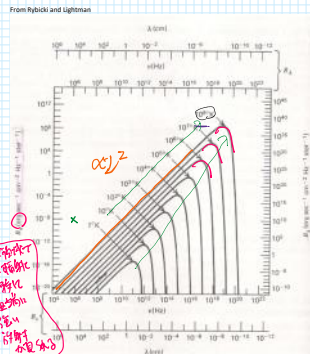
$$3(e^{h\nu/kT}-1) - \nu^3 e^{h\nu/kT} \cdot \frac{h}{kT} = 0$$

$$3e^{h\nu/kT} - 3 - e^{h\nu/kT} \cdot \frac{h\nu}{kT} = 0$$

$$(3 - \frac{h\nu}{kT}) e^{h\nu/kT} = 3$$

$$\frac{h\nu}{kT} \approx 2.82$$

$$h\nu = 2.82kT \approx 3kT$$



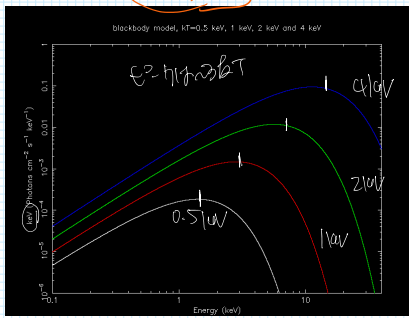
電荷体から放射される電磁波の強度は温度の4乗で増加する

黒体放射のピークは温度の逆数で移動する

Effective temperature of 3kT

λ (cm)	Epoch (1)	GBT (2)	P.A. (10° East, GA) (3)	SNR (4)	PA (5)	SNR (6)	PA (7)	SNR (8)	PA (9)	T <sub>mb</sub> (10 <sup>2</sup> K) (10)	T <sub>mb</sub> (10 <sup>2</sup> K) (11)	T <sub>mb</sub> (10 <sup>2</sup> K) (12)
1.3	2013-04-02	GM, V77	166.7 ± 0.6	7.0	-2	9.6	3.4	126 ± 22	36	14	5.3	12
4.2	2013-12-30	At, V1	205.1 ± 0.5	10	18.6	4.3	125 ± 11	142	13	12	5.2	18
15	2013-04-04	At, V1	157.0 ± 0.7	10	11.6	4.0	124 ± 19	122	13	12	5.2	18
18	2013-04-04	At, V1	157.0 ± 0.7	10	8.9	2.0	125 ± 11	127	13	12	5.2	18
18	2013-04-25	At, V1	171.0 ± 0.5	10	-38	12.0	3.0	124 ± 24	144	14	6.3	18

XSPEC



4. Calculate energy density of the blackbody emission

Use the following formula:  $u = \frac{4\pi}{c} \int_0^\infty B_\nu(T) d\nu$  [ergs/cm<sup>3</sup>]

$$u = \frac{4\pi}{c} \int_0^\infty B_\nu(T) d\nu = \frac{8\pi}{c} \frac{kT^4}{15 h^3} \int_0^\infty \frac{x^3}{e^x - 1} dx$$

$$= \frac{8\pi^5}{15 h^3 c^3} T^4 = 7.56 \times 10^{-15} (T/\text{K})^4 \text{ [ergs/cm}^3\text{]}$$

5. Estimate the energy density of the 2.7K cosmic microwave background radiation. Compare this with a typical interstellar magnetic energy density with  $B \approx 5 \mu\text{T}$

$$7.56 \times 10^{-15} \cdot (2.7)^4 \text{ [ergs/cm}^3\text{]} \approx 4 \times 10^{-13} \text{ [ergs/cm}^3\text{]}$$

$$1 \text{ eV} \approx 1.6 \times 10^{-12} \text{ erg} \Rightarrow 0.25 \text{ [eV/cm}^3\text{]} \approx 0.2 \text{ [eV/cm}^3\text{]}$$

$$\frac{1}{8\pi} (B/\text{Gauss})^2 = \frac{1}{8\pi} (5 \times 10^{-6})^2 \approx 0.2 \times 10^{-12} \text{ erg/cm}^3 \approx 0.2 \text{ [eV/cm}^3\text{]}$$

6. Explain the difference of the effective temperature, color temperature, and brightness temperature

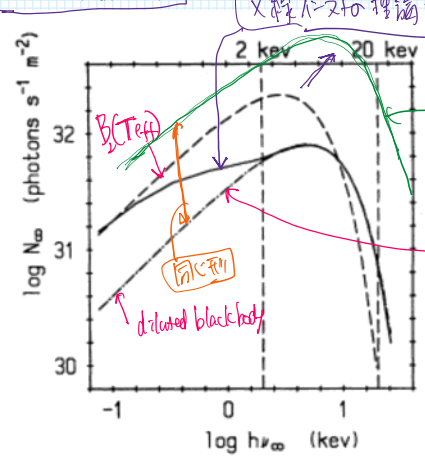
有效温度 (brightness temperature) vs 色温度 (color temperature) vs 黒体放射 (blackbody radiation)

黒体放射の存在を示す ⇒ 3kT 程度を示す

7. Draw the black body spectrum with the temperature E=kT in a wide energy range using the ν f\_ν plot. Observe the energy spectral distribution around kT, if it is widely distributed, or narrowly distributed.

Are there any continuum spectra which are more concentrated (narrowly distributed) around kT? はい!

Boisard 1987, PASJ, 39, 287 X-ray energy spectrum of X-ray burst



$$L = 4\pi R^2 \sigma T_{\text{eff}}^4$$

有効温度

$$L = 4\pi R^2 \int f(\nu) d\nu = 4\pi R^2 \int \frac{B_\nu(T_{\text{eff}})}{4\pi D^2} d\nu = 4\pi R^2 \left(\frac{T_{\text{eff}}}{T_{\text{bb}}}\right)^4$$

中性恒星表面 光学的に厚い降着円盤

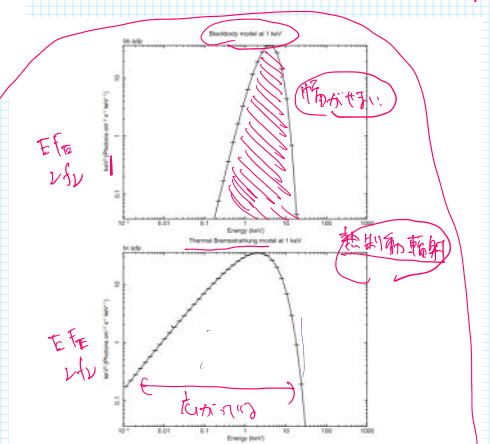
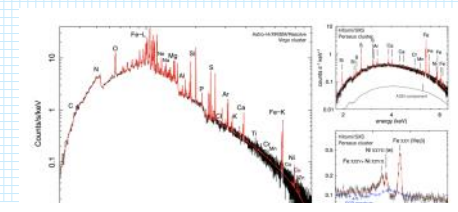


Figure 5.4: Blackbody spectrum with  $kT = 1 \text{ keV}$  (top) and thermal bremsstrahlung spectrum with the same temperature (bottom) in the units of  $\text{keV}^2/\text{s}/\text{cm}^2/\text{keV}$ . Both have peaks at around  $3 kT$  (section 5.3.2). We can see thermal bremsstrahlung is much wider and particularly extends toward lower energies.

https://www.osti.gov/jat/2003.04662.pdf XRISM white-paper Thermal plasma spectra from clusters of galaxies



2. Thin-thermal plasma emission

1. From what kinds of sources will we observe thin-thermal X-ray emission?

4πr  
- 湯皮

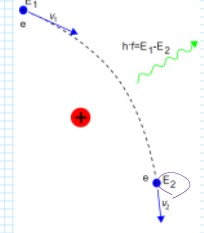
$$B_{\lambda}(T_{\text{in}}) dV$$

$$\frac{\int B_{\lambda}(T_{\text{in}}) dV}{\sigma T_{\text{in}}^4}$$

$$\underline{4\pi R^2 \sigma T_{\text{eff}}^4}$$

2. Explain the mechanisms of the continuum emission from thin-thermal plasmas

<https://upload.wikimedia.org/wikipedia/commons/7/7e/Bremstrahlung.svg>



3. If the hot plasma is only composed of hydrogen ( protons and electrons), what kind of spectra are expected from thin-thermal plasma?

4. Indicate a simple approximate formula of the thermal bremsstrahlung spectrum.

5. Indicate examples of X-ray energy spectra from thin-thermal plasmas with different temperatures (say, from  $kT=1$  keV to 20 keV). Observe changes of the energy spectra with temperatures paying attention to the following points:

- a. Emission lines
- b. Slope of the continuum in 1 - 10 keV
- c. Peak energy of the continuum

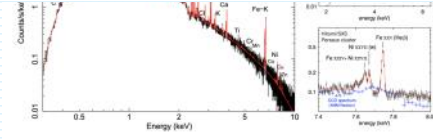


Figure 7. Left: XSPEC simulated spectrum (black) of the core of M87 as observed with  $100 \times$  and the best-fitting model (red). Adopted from [1]. Right: Observed spectrum from the core of the Perseus cluster. Broad-band spectrum is shown on the top panel (note that the gate value was in place during the observation and it also has X-ray below  $\sim 2$  keV), while the zoomed-in on the high-energy end spectrum is shown in the bottom panel. The best-fitting model is shown in red. Different resolved individual lines in the spectrum (black vs. blue points) [1].

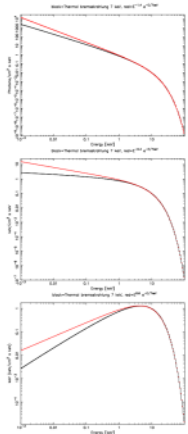


Figure 5.3. Comparison of  $kT=7$  keV thermal bremsstrahlung spectra (black, brems model in xspec) and a cut-off power-law model  $\propto E^{-\alpha} \exp(-E/kT)$  where  $kT=7$  keV (red). From top to bottom, unit of the Y-axis is  $[\text{photons}/\text{keV}/\text{cm}^2/\text{s}]$ ,  $[\text{keV}/\text{keV}/\text{cm}^2/\text{s}]$  and  $[\text{keV}^2/\text{s}/\text{keV}/\text{cm}^2]$ , and the  $\alpha$  value is 1.4, 0.4,  $-0.6$ , respectively.

<https://www.iss.uva.nl/home/obscasabib/obscasim/TTEP/IMG/2010/Note.pdf>

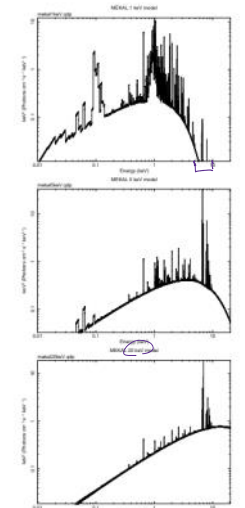


Figure 5.5. Theoretical energy spectra from thermal plasmas at 1 keV, 5 keV and 20 keV. The "mask" model in XSPEC is used. We can see that, as the temperature increases, (1) the continuum peaks shift toward higher energies, and (2) emission lines from heavier elements are more prominent.

3. Thermal Comptonization

- 1. Observed how the "cut-off power-law" spectra are formed by thermal (inverse) Comptonization process
- 2. Observe how the spectral shape changes with the scattering optical depth
- 3. In which objects the thermal comptonization spectra are expected?

Thermal comptonization model  
 Pozdnyakov et al. (1983)  
<https://arxiv.org/abs/astro-ph/8301017>  
[https://inspirehep.net/literature/1983ASPRv\\_2\\_189D.pdf](https://inspirehep.net/literature/1983ASPRv_2_189D.pdf)

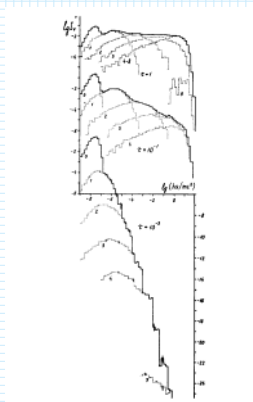


Figure 28. Comptonization of low-frequency radiation in a cloud of weakly relativistic electrons for three values of the optical depth  $\tau$ . Dashed curves represent the contribution of the separate scattering events. The successive scatterings by Maxwellian electrons continue to form a power-law spectrum for the hard tail side.

Longair, "High Energy Astrophysics"

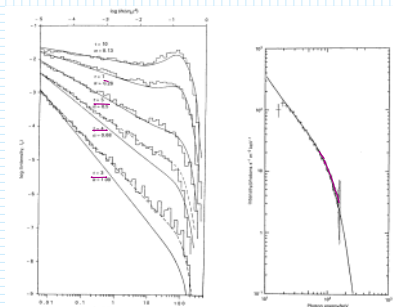


Figure 4.11. The Comptonization of low-frequency radiation in a spherical plasma cloud having  $kT = 100$  eV. The curves are similar to those of the Comptonization spectrum in a spherical plasma cloud, but the optical depth  $\tau$  is the Comptonization optical depth  $\tau_{\text{comp}} = 2.8 \times 10^{21} n_e R$ , where  $n_e$  is the electron density and  $R$  is the radius of the cloud.

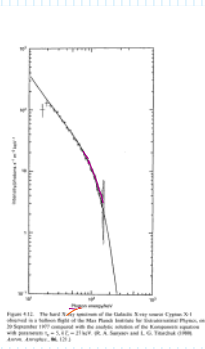


Figure 4.12. The hard X-ray spectrum of the Galactic X-ray source Cygnus X-1 obtained in a hybrid mode in the X-ray band by the International X-ray Observing and Information Center (IXO) in 1975. The solid line is the best-fitting power-law model with parameters  $\alpha = 1.17 \pm 0.02$  and  $E_{\text{cut}} = 1.0 \pm 0.1$  MeV. The dashed line is the model with parameters  $\alpha = 1.17 \pm 0.02$  and  $E_{\text{cut}} = 1.0 \pm 0.1$  MeV.

<https://link.springer.com/content/pdf/10.1007/s00158-007-0006-3.pdf>

A widely accepted model for Galactic Black hole binaries

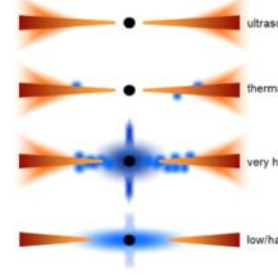
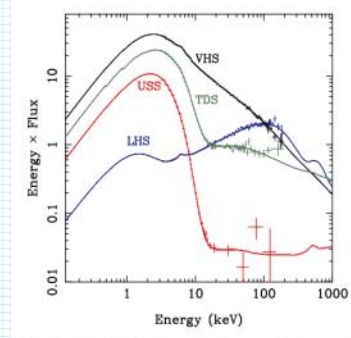
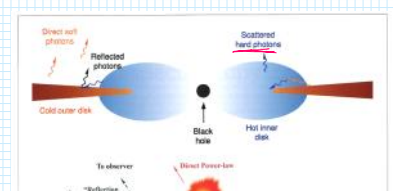


Fig. 9 The left hand panel shows a selection of states taken from the 2005 outburst of GRO J1655-40. The right hand panel shows the proposed accretion flow changes to explain these different spectral states differing contributions from the disc, hot inner flow and its associated jet, active regions above the a wind

AGN accretion geometries under debate  
 Across the Universe, Research at the Nicolaus Copernicus Astronomical Center, Warsaw 2020

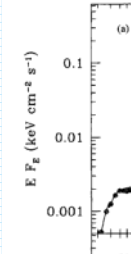
Andrzej A. Zdziarski, Barbara De Marco

Sombros and lampposts: The geometry of accretion onto black holes



<https://ul.adsabs.harvard.edu/abs/21396>

Thermal comptonization model for



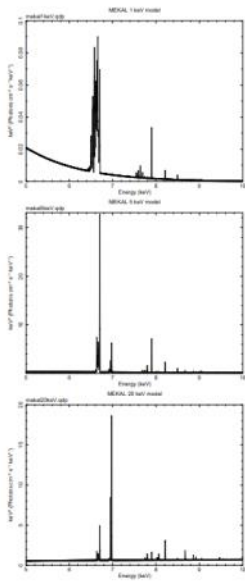
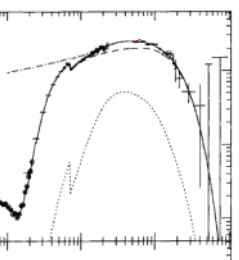


Figure 5.6: Expansion of the iron K-line region in the previous figure. Iron line missions from the 1 keV, 5 keV and 20 keV thermal plasmas are indicated. Around 6.6 - 6.7 keV, FeXXV (He-like) lines are seen, and around 7.0 keV, FeXXVI (H-like) lines are observed. As the plasma temperature increases, lines from more highly ionized ions are observed.

dominant

1655-40.  
tra, with  
disc and

MNRAS 283, 1637-1646 (2005)  
VG4151



し

#### 4. Non-thermal emission

Often non-thermal **particle acceleration** takes place in the universe (shocks, magnetic acceleration etc.), and each **relativistic** electron ( $v \sim c$ ) has the energy  $E = mc^2\gamma$ , where  $\gamma = 1/\sqrt{1 - (v/c)^2} \gg 1$ .

The electron energy distribution  $N(E)$  often becomes a power-law, as

$$N(E)dE \propto \gamma^{-p} d\gamma.$$

In the case of **synchrotron emission** or **relativistic inverse-Compton scattering**, typical photon frequency  $\nu$ , from a single electron with the energy  $E = mc^2\gamma$  is known to be proportional to  $\gamma^2$ .

By integrating over the electron energy distribution, synchrotron spectra or relativistic inverse-Compton spectra may be expressed as

$$F(\nu) \propto \int S(\nu/\nu_c) \gamma^{-p} d\gamma.$$

1. Indicate the synchrotron emission or relativistic inverse-Compton scattering spectra become power-law

$$\propto \nu^{-s}$$

with the index

$$s = \frac{p-1}{2}$$

2. Which sources are expected to emit non-thermal Comptonization and/or synchrotron emission?

3. When the relativistic electrons, soft input photons (energy density  $U_{ph}$ ), and magnetic fields (energy density  $U_B$ ) coexist, both the synchrotron emission and inverse-Comptonization take place simultaneously.

In the case of the SNR RXJ1713.7-3946, observe that the following relationship holds between the synchrotron luminosity ( $P_{syn}$ ) and the inverse-Comptonization luminosity ( $P_{comp}$ ).

$$\frac{P_{synch}}{P_{compt}} = \frac{U_B}{U_{ph}}$$

4. In Fig. 19 of Aharonian et al. 2006, the relativistic electron energy distribution has been assumed to be  $\gamma^p \exp(-m^2c^2/\epsilon)$ , where  $p$  is 2 and  $\epsilon_0 = 100$  TeV.

Explain the following values semi-quantitatively:

- (a) Inverse Compton spectral peak energy
- (b) Cut-off energy of the synchrotron spectrum and the inverse Compton spectrum
- (c) Slope of the synchrotron spectrum and the inverse Compton spectrum below the cut-off energies

Synchrotron and Inverse Compton model for the supernova remnant RXJ1713.7-3946

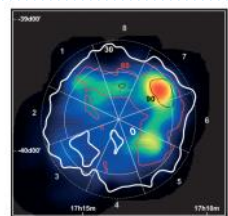


Fig. 18. ASCA X-ray (1-3 keV band, Uchiyama 2005) image of RX J1713.7-3946, overlaid with contours of the smoothed, acceptance-corrected HESS gamma-ray image. The colored contour levels are labeled and intensity scaled at 30, 60, and 90 counts. There are eight gray thin lines are eight wedge-shaped regions for which the radial profiles are compared to each other in Fig. 16. Note that in the ASCA image, most of the regions (thin solid lines) do not reach as far as in the HESS image (thin dashed lines), accounting for the limited field of view of ASCA, whose coverage did not always extend to the boundaries of the SNR. As explained in the main text, the ASCA image was smoothed to match the HESS point-spread function to enable comparison of the two images.

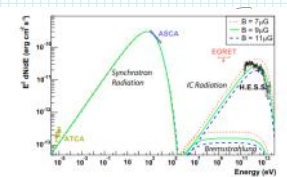


Fig. 19. Broadband SED of RX J1713.7-3946. The ATCA radio data and ASCA X-ray data (Hiraga 2005) for the whole SNR are indicated, along with the HESS measurement and the EGRET upper limit. Note that the radio flux was determined in Lazic et al. (2004) for the northwest part of the shell only and was scaled up by a factor of two here to account for the whole SNR. The synchrotron and IC spectra were modelled assuming a source distance of 1 kpc, an age  $T$  of 1000 years, a density  $n$  of  $1 \text{ cm}^{-3}$ , and a production rate of relativistic electrons by the acceleration mechanism in the form of a power law of index  $p \approx 2$  and an exponential cutoff of  $E_0 = 100 \text{ TeV}$ . Shown are three curves for three values of the mean magnetic field, 7  $\mu\text{G}$ , 9  $\mu\text{G}$ , and 11  $\mu\text{G}$ , to demonstrate the required range of the B field strength for this scenario. The electron luminosity is adopted such that the observed X-ray flux level is well matched. For the three magnetic field values the luminosity  $L_e$  is  $1.77 \times 10^{47} \text{ erg s}^{-1}$  (7  $\mu\text{G}$ ),  $L_e = 1.14 \times 10^{48} \text{ erg s}^{-1}$  (9  $\mu\text{G}$ ), and  $L_e = 0.81 \times 10^{49} \text{ erg s}^{-1}$  (11  $\mu\text{G}$ ).

<https://arxiv.org/abs/101089.0004-637X/773/2/138.pdf>

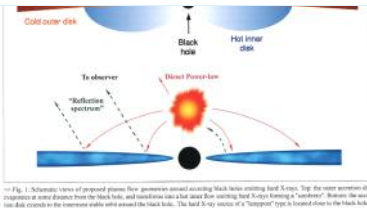


Fig. 1. Schematic view of proposed plasma flow geometry around rotating black hole emitting hard X-rays. Top: The outer accretion disk progresses as more distance from the black hole, and transforms into a hot flow emitting hard X-rays forming a "magnetosphere". Below: the magnetosphere extends to the innermost stable orbit around the black hole. The hard X-ray source of a "hotspot" type is located close to the black hole.

Miyakawa, Ebisawa and Inoue (2012), PASJ, 64, 140

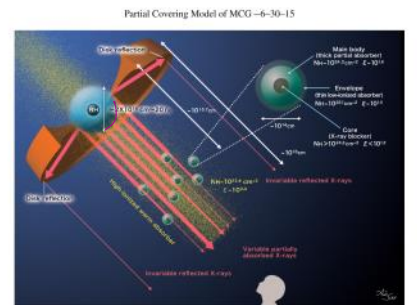
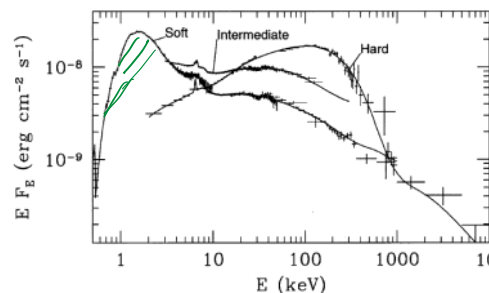


Fig. 12. Schematic picture of the variable partial-covering model for MCG-6-30-15 and the internal structure of the absorbing cloud.

Thermal Compton & non-thermal Compton model for Cyg X-1



Synchrotron and Inverse Compton model for Crab pulsar

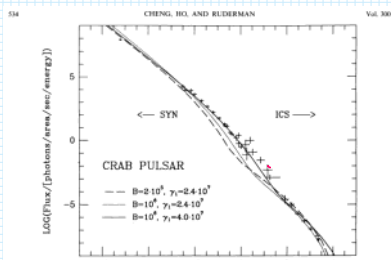


Fig. 4. Calculated and observed spectra of the pulsed electromagnetic emission from the Crab pulsar. The theoretical spectra use the parameters discussed in § 4.1.  $B = 4 \times 10^8 \text{ G}$  and  $\gamma = 10^3 \text{ GeV}$ . The observed and modelled emission spectra are shown for  $B = 7 \mu\text{G}$  and  $B = 2.1 \times 10^8 \text{ G}$  and  $\gamma = 10^3 \text{ GeV}$ . The solid curve is that for  $B = 10^8 \text{ G}$  and  $\gamma = 10^3 \text{ GeV}$ . All the spectra are normalized to be the total observed intensity. As in § 4.1, the observed emission is reduced by the absorption due to the magnetosphere. The dashed curves are calculated from inverse Compton scattering of the outer energy conversion photons by the secondary pairs which flow in opposite from the dipole flow (theoretical spectrum) or above the dipole flow (observed spectrum). The observed emission is compared to each other. Other values from the H.E.S.S. Collaboration (2006), Aharonian et al. (2007), Engel (1982), Matsumura, Uchiyama, and Matsumura (1982).

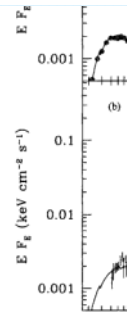
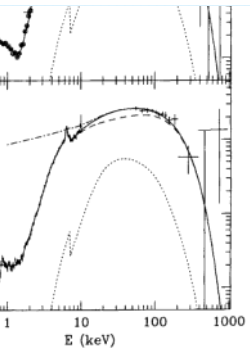
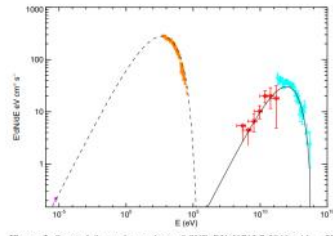


Figure 2. The X-ray spectra of MCG-6-30-15 obtained by ROSAT ASCA and OSSE. The plot shows the Comptonization spectrum, the soft X-ray Comptonization spectrum, and the soft X-ray Comptonization spectrum. The dot-dash curves are given in Table 1, and the upper curves with circles. The data are added for plotting.



Spectra of NGC 4151 as observed (a) in 1991 by *Ginga* and OSSE, and (b) in 1993 May by *ROSAT*. The dashed curves give the absorbed thermal component, the dotted curves give the reflection component, the dash-dotted curves give the models of the observed continuum, and the solid curves give the unabsorbed continuum plus the  $K\alpha$  line. The model parameters are listed in Table 1. The plotted data have been rebinned for clarity. The *ROSAT* data are marked with open circles. The  $2\sigma$  upper limits are shown. The *ROSAT* data are marked with open circles. The *ASCA* data from all four *ASCA* detectors have been co-added.



**Figure 3.** Spectral fit to observations of SNR RX J1713.7-3946 with a 2D MHD simulation. The different emission processes are synchrotron (dashed line), IC (solid line). The data are from Acero et al. (2009; in radio), Tanaka et al. (2008; Suzaku X-ray), Abdo et al. (2011; Fermi-LAT), and Aharonian et al. (2006; HESS). Note that the two lowest energy Fermi-LAT points are upper limits. The seed photon field in the IC process includes an IR component with a temperature of  $T = 30$  K and an energy-density of  $1.2 \text{ eV cm}^{-3}$  and the microwave background radiation (Li et al. 2011).



