Lesson 5 (2021-05-11) : What more to know about black holes

2021年4月20日 16:42

Gravitational energy release from black holes

1. **Innermost Stable Circulr Orbits (ISCO) of non-rotating blackhole (Schwarzschild black hole)?**
   What about a spinning black hole at the maximum rate (extreme Kerr black hole)?

$a=0$

$a=1$

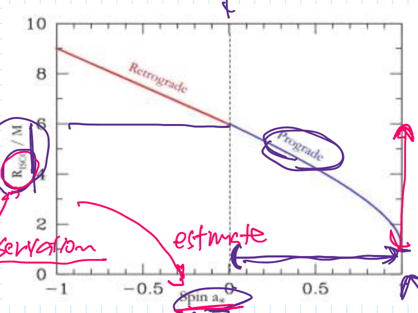
$r=\infty$, $E=0$

figure from 10.1007/s11433-018-9297-0

$$R_{ISCO} = 3R_S = 6R_G$$

$$'' = 0.5R_S = R_G$$

$$R_G = \frac{GM}{c^2} \qquad c=M=1 \qquad R_G = M$$



2. **Ignoring general relativity, estimate total energy E (potential energy + kinetic energy) of a mass m rotating at the ISCO around a black hole with mass M. Consider the Schwartzshild case and the extreme-Kerr case.**

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$E = \frac{1}{2}mv^2 - G\frac{Mm}{r}$$

$$= -\frac{GMm}{2r} = \begin{cases} -\frac{1}{2}\cdot\frac{GMm}{6\frac{GM}{c^2}} = -\frac{1}{12}mc^2 \ (a=0) \\ -\frac{1}{2}\cdot\frac{GMm}{\frac{GM}{c^2}} = -\frac{1}{2}mc^2 \ (a=1) \end{cases}$$

0.083, approximate

$1-\sqrt{8/9} = 0.057$

$1-\sqrt{1/3} = 0.42$

precise

3. **When the mass m reaches the ISCO from infinisty (where the initial velocity is assumed to be zero), the energy -E is released. What will be the energy conversion efficiency, η, where -E = ηmc²? Compare with the precise values using general relativity.**

$\eta = 0.057 \ (a=0)$

$= 0.42 \ (a=1)$

4. **Compare with the efficiency of nuclear burning.**

$H \rightarrow Fe$ $\quad \eta = 0.009$

Black hole spin

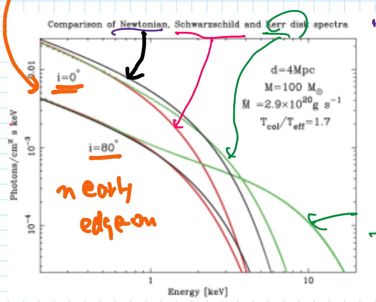
1. **How can we estimate black hole spin from X-ray observation?**

X-ray energy spectra of accretion disk → ISCO ← Mass (dynamical motion), spin

· distance to the source

· inclination

face-on (正面)



Ebisawa et al. (2003)

Measuring the spins of accreting black hole

dynamical motion → M_{BH}

Sun Mercury

Table 1. Spin results to date for eight black holes[a].

Measuring the spins of accreting black hole

d-namical motion

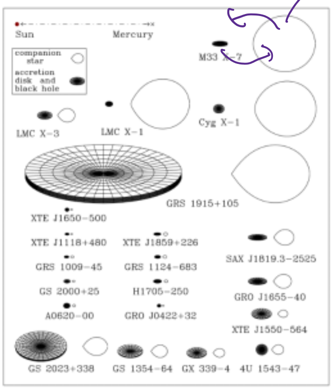


Table 1. Spin results to date for eight black holes[a].

	Source	Spin a_*	Reference
1	GRS 1915+105	>0.98	McClintock *et al* 2006
2	LMC X-1	$0.92^{+0.05}_{-0.07}$	Gou *et al* 2009
4	M33 X-7	0.84 ± 0.05	Liu *et al* 2008, 2010
3	4U 1543−47	0.80 ± 0.05	Shafee *et al* 2006
5	GRO J1655−40	0.70 ± 0.05	Shafee *et al* 2006
6	XTE J1550−564	$0.34^{+0.20}_{-0.28}$	Steiner *et al* 2010b
7	LMC X-3	<0.3[b]	Davis *et al* 2006
8	A0620−00	0.12 ± 0.18	Gou *et al* 2010

[a] Errors are quoted at the 68% level of confidence.
[b] Provisional result pending improved measurements of *M* and *i*.

Gravitational Wave and black holes

1. **What happens if two "bare" black holes (i.e., no accretion disks) merge in the *Binary Black Hole Coalescence*?**

GW is emitted   EM is not emitted

Nobel prize 2017

2. **Let's assume that we detected a gravitational-event due to a blackhole merger, where amplitude of the gravitational wave is 10^{-21}. How much the distance between Sun and Earth (1 astronomical unit) varies due to this gravitational event? Answer with the unit of Bohr radius.**

0.5A $= 0.5 \times 10^{-8}$ [cm]

1 A.U. 500 lt-sec $= 500 \times 3 \times 10^{10}$ [cm] $= 1500 \times 10^{10}$ [cm] $= 1.5 \times 10^{13}$ [cm]

$1.5 \times 10^{13} \times 10^{-21}$ [cm]

$= 1.5 \times 10^{-8}$ [cm]

≒ 3 Bohr radius

3. **What about in the case of *neutron star mergers*?**

GW and EM are emitted

The historical first neutron star merger paper

These observations support the hypothesis that GW170817 was produced by the merger of two neutron stars in NGC 4993 followed by a short gamma-ray burst (GRB 170817A) and a kilonova/macronova powered by the radioactive decay of *r*-process nuclei synthesized in the ejecta.

Binary Neutron Star Mergers as the Production Site of Gold, Platinum, and Rare Earth Elements

4. **What do you expect in the case of the *binary super-massive black hole merger*, where both of the super-massive black holes are X-ray active AGN (i.e., they have accretion disks)?**

"A unique experiment to explore black holes" by ESA

See also the You tube video by ESA

Hawking radiation and black hole evaporation

1. **According to Hawking (1974), a black hole with the mass M has a temperature**

$$T = \frac{\hbar c^3}{8\pi k G M},$$

Estimate the temperature in [K] for stellar mass black holes. Would it be possible to detect the "Hawking ratiation" from these black holes?

2. **Black holes may evaporate at a timescale of**

$$\tau \approx 400\left(\frac{M}{10^{10}\ \mathrm{g}}\right)^3 \mathrm{s}.$$

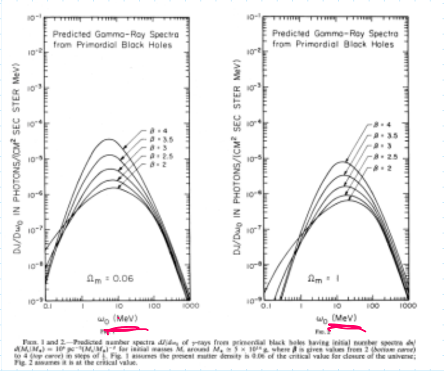
Carr et al. (2010)

Estimate the black hole mass which would evaporate in the age of the Universe

3. **Estimate the temperature (in eV) of those "primordial black holes" which may have been created in the early universe and would evaporate in the age of the universe.**

4. **How can we search for such primordial black holes?**

Gamma-rays from primordial black holes



Search for Gamma-ray emission from local primordial black holes with the Fermi Large Area Telescope

Abstract

Black holes with masses below approximately 10^{15} g are expected to emit gamma-rays with energies above a few tens of MeV, which can be detected by the *Fermi* Large Area Telescope (LAT). Although black holes with these masses cannot be formed as a result of stellar evolution, they may have formed in the early universe and are therefore called primordial black holes (PBHs).