Lesson 4 (2021-04-27) : What to know about black holes

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Basic of Black Holes:

1. **In Newtonian mechanis, obtain the radius of a star with mass M, where the escape-velocity is equal to the light velocity. Compare this with the Schwarzschild radius.**

$E = \frac{1}{2}mv^2 - G\frac{Mm}{r} = 0 = \frac{1}{2}m v_{esc}^2 - \frac{GMm}{R}$

$v_{esc} = \sqrt{\frac{2GM}{R}}$

$v_{esc} \to c$

$r = \infty,\ v = 0,\ E = 0$

$c = \sqrt{\frac{2GM}{R}}$

$R = \frac{2GM}{c^2}$ ↕ agree!! R_S

2. **Maximum known mass of "stellar" black holes? (See these article and paper by LIGO collaboration)**

GRS1915+105 $M \approx 14 M_\odot$ ← X-ray source

$\sim 80 M_\odot$ ← Gravitational Wave (BH merger)

3. **Mass of the black hole in the center of our Galaxy (Sgr A*)? (A latest result, Stellar Orbits around the black hole, Nobel Prize 2020!)**

$\sim 4\times10^6 M_\odot$ ← precise!!

4. **Maximum known mass of super-massive black holes (AGN, quasars)?**

$\sim 10^{10} M_\odot$ ← approximate

5. **Are there "intermediate mass black holes (IMBHs)" with a mass of 100-1000 Msolar? Which sources are the candidates of the intermediate mass black holes?**

ULXs → Not sure!!

Ultraluminous X-ray Sources

A 400-solar-mass black hole in the galaxy M82 ?

Maybe $\sim 20 M_\odot$ BH shining at super-Eddington luminosity

remember

Feel the "size" of Black Holes:

1. **Estimate apparent size of a black hole, dividing the Schwarzschild radius by the distance to the source. Which is easier to "resolve", stellar black hole in our Galaxy, or super-massive black holes in other galaxies?**

$\Delta\theta \simeq \frac{R_S}{d} = \frac{2GM/c^2}{d} = \frac{3\,\mathrm{km}\,(M/M_\odot)}{d} = \frac{30\,\mathrm{km}\,(M/10M_\odot)}{(d/10\,\mathrm{kpc})\,10\,\mathrm{kpc}}$

$1\,\mathrm{pc} \simeq 3\times10^{18}\,\mathrm{cm}$

$\simeq \frac{30\,\mathrm{km}\,(M/10M_\odot)}{10\times10^3\times3\times10^{18}\,\mathrm{cm}\,(d/10\,\mathrm{kpc})} = 10^{-16}\,[\mathrm{rad}]\,\frac{(M/10M_\odot)}{(d/10\,\mathrm{kpc})}$

$1\,\mathrm{rad} = \frac{180^\circ}{\pi} = \frac{(180\times3600)''}{\pi} = 2\times10^5\,[\mathrm{arcsec}]$

$\Delta\theta \simeq 2\times10^{-11}\,(\mathrm{arcsec})\,\frac{(M/10M_\odot)}{(d/10\,\mathrm{kpc})}$ Stellar BH

Sgr A $(M \simeq 4\times10^6 M_\odot,\ d \simeq 8\,\mathrm{kpc})$

$\Delta\theta \simeq 2\times10^{-11}\cdot\frac{4\times10^5}{0.8} = 10^{-5}\,[\mathrm{arcsec}]$ SMBH

2. **Estimate spatial resolution of an radio interferometer at 1mm, where the base-line is 10,000 km (=maximum on Earth). Also, estimate spatial resolution of an X-ray interferometer at 1 A, with the base-line is 10 m. Which has better spatial resolution?**

$\Delta\theta \simeq \frac{\lambda}{D} \simeq \frac{1\,\mathrm{mm}}{10^4\,\mathrm{km}} \simeq 10^{-10}\,(\mathrm{rad}) \simeq 20\,(\mu\,\mathrm{arcsec})$

Event horizon telescope ← On the earth

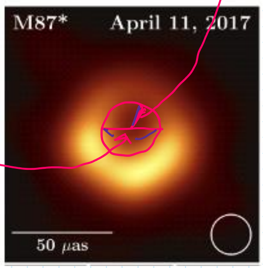
HALCA - The first Space VLBI project ← from Space

Micro-arcsecond X-ray Imaging Mission → only dream!!

HALCA - The first Space VLBI project

Micro-arcsecond X-ray Imaging Mission

3. According to General Relativity, "Photon-capture radius" of a black hole is $\sqrt{27}\ r_g$, where r_g is the gravitational radius ($=GM/c^2$). In 2020, the Event Horizon Telescope observed a bright photon "ring" around central blackhole M87 (distance = 16.8 Mpc), where the diameter was 42 μas. By identifyng the ring radius as the photn-capture radius, estimate the black hole mass. (see 2019ApJ...875L...1E)



4. Assume that black hole is a sphere having the Schwarzschild radius (ignore General relativistic effects). Estimate "density" of the black hole, simply dividing the mass by the volume. Is the density smaller or larger for more massive black holes?

5. Can the black hole "density" smaller than that of water? If yes, when?

6. Estimate light-crossing time of the Schwarzschild radius of a black hole with mass M.

7. Which is technically easier to detect X-ray variability taking place near the Schwarzschild radius, stellar mass black holes or super-massive black hole?