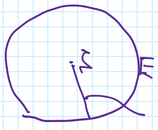
Lesson 3 (2021-04-20) : Basic Astronomy and X-ray astronomy

2021年4月13日　7:00

Remember or derive the following numbers/formulae which often appear in astrophysics

1. Value of 1 A.U. (astronomical unit), together with its definition



1A.U. ← remember

1.5×10^{8} [km]

2. 1 A.U. in light-seconds

← 1A.U. ≒ 500 lt-sec

$c \fallingdotseq 3\times10^{10}$ [cm/s]

$= 500\,(\text{sec}) \times 3\times10^{10}$ [cm/s]

$= 1500\times10^{10}$ [cm]

$= 1.5\times10^{13}$ [cm]

3. 1 year in seconds

$365\times24\times3600\,(\text{sec}) = 3.15\times10^{7}$ (sec)

$\simeq \pi\times10^{7}$ (sec)　π千万秒

4. Age of the Universe in year

13.8×10^{9} year.　138億年 ← Planck

5. Age of the Universe in second

$13.8\times10^{9}\times3.15\times10^{7}$ [sec] $= 4\times10^{17}$ [sec]

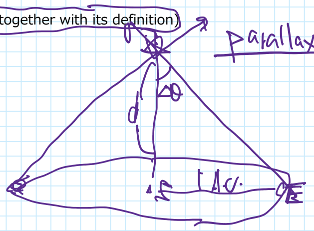
Is there a clock which is precise with the accuracy within a second in the age of the universe?

Link to Prof. Katori's lab page

6. What is the scale of the universe (ratio between the minimum/maximum length or the longest/shortest time)?

$$\frac{\text{Age of the Universe}}{\text{Planck time}} \simeq \frac{\text{Horizon of the universe}}{\text{Planck scale}} \simeq \frac{4\times10^{17}\,(\text{sec})}{5\times10^{-44}\,(\text{sec})} \simeq 10^{61}$$

7. 1 pc (parsec; together with its definition)



parallax (視差)

$d\cdot\Delta\theta \simeq 1$ A.U.

when $\Delta\theta = 1''$　$d = 1$ parsec (pc)

$1'' = \left(\frac{1}{3600}\right)^{\circ} = \frac{1}{3600}\cdot\frac{\pi}{180}$ [rad]

$1^{\circ} = 60'$

$1' = 60''$

$$1\text{pc} = \frac{1.5\times10^{13}\,[\text{cm}]}{\frac{1}{3600}\cdot\frac{\pi}{180}} = 3.09\times10^{18}\,[\text{cm}]$$

$1\text{pc} \fallingdotseq 3\times10^{18}$ [cm]

About Compact objects (black Holes, neutron stars and white dwarfs)

1. Schwarzschild radius of a star with mass *M*.

$R_s = \frac{2GM}{c^2}$　remember!!

G.R. → Einstein equation

metric

Schwarzschild solution

2. Schwarzschild radius of Sun and Earth.

$\frac{2GM_{\odot}}{c^2} \simeq 2.95$ km $\simeq 3$ km

$\frac{2GM_{\oplus}}{c^2} \simeq 9$ mm

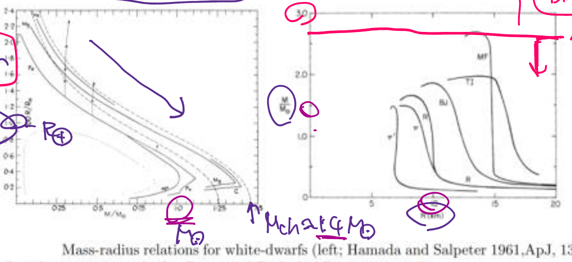
3. Maximum mass of a white-dwarf (Chandrasekhar limit).

$\sim 1.4\,M_{\odot}$

Nobel prize in 1983

electron degenerate pressure

縮退圧 $R_{\oplus}$



$M_{\odot}$

$M_{ch} \simeq 1.4\,M_{\odot}$

Black holes!!

maximum neutron star mass

M/R

Mass-radius relations for white-dwarfs (left; Hamada and Salpeter 1961,ApJ, 134, 683) and neutron stars (right; Baym and Pethick ARAA 1979, 415).

4. Typical mass and radius of a white-dwarf?

$R_{\odot} = 696340$ km $\simeq 100\,R_{\oplus}$

$R_{\oplus} = 6371$ km

5. Theoretical maximum mass of a neutron star (≈ minumum mass of a black hole)?

neutron degenerate pressure

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neutron degenerate pressure

$\boxed{\sim 3 M_\odot}$ ← remember!!

6. Typical mass and radius of a neutron star?

$M \simeq 1.4 M_\odot$

$R \simeq 10\,\mathrm{km}$

7. Compare typical radius of a neutron star and a white dwarf with their Schwarzschild radius

N.S. $M \simeq 1.4 M_\odot \rightarrow R_S \simeq \frac{2GM}{c^2} = \frac{2GM_\odot}{c^2}\frac{M}{M_\odot} \simeq 3\,\mathrm{km} \cdot 1.4 \simeq 4\,[\mathrm{km}]$

W.D. $M \simeq 1 M_\odot \rightarrow R_S \simeq 3\,\mathrm{km}$ Radius $\sim 6000\,\mathrm{km}$

8. Are heavier white-dwarfs larger or smaller in size? What about neutron stars? → $2000\,R_S$

9. Typical central density of a neuron star?

$\sim 10^{15}\,[\mathrm{g/cm^3}]$

10. Typical magnetic field of an X-ray pulsar (in gauss)? What about extremely strongly magnetized pulsars (**magnetors**)

$\sim 10^{12}\,\mathrm{G}$ $\sim 10^{15}\,\mathrm{G}$

11. How can we observationally constrain the equation of state and study interior of neutron stars?

NICER page at NASA

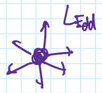
Eddington Limit

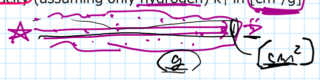
e⁻

p

$\sigma_T = 6.65 \times 10^{-25}\,[\mathrm{cm^2}]$

1. Thomson scattering opacity (assuming only hydrogen) κ_T in $[\mathrm{cm^2/g}]$





$[\mathrm{cm^2/g}]$

$m_H \simeq m_p \simeq 1\,\mathrm{GeV}/c^2$

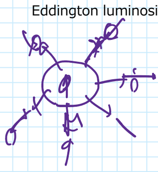
$\approx 10^9 \cdot 1.6 \times 10^{-12}\,\mathrm{erg}/(3 \times 10^{10}\,\mathrm{cm/s})^2$

$\doteqdot 1.8 \times 10^{-24}\,[\mathrm{g}]$

$= 1.67 \times 10^{-24}\,[\mathrm{g}]$

$$\kappa_T \simeq \frac{6.65 \times 10^{-25}\,[\mathrm{cm^2}]}{1.7 \times 10^{-24}\,[\mathrm{g}]} \simeq \boxed{0.4\,[\mathrm{cm^2/g}]}$$

2. Derive the formula of the Eddington luminosity of a star with mass *M*, and the Eddington luminosity of a solar-mass star.



erg/s/cm²

$$\boxed{\frac{G m_H M}{r^2}} = \boxed{\frac{\mathrm{Flux} \cdot \sigma_T}{c}} = \frac{1}{c}\frac{L_{Edd}}{4\pi r^2} \cdot \sigma_T$$

$$\frac{c\,4\pi G m_H M}{\sigma_T} = L_{Edd}$$

$\sim 1.4 M_\odot$

3. What is the blackbody temperature of a neutron star with a radius of 10 km? What about a white-dwarf shining at the Eddington limit with the radius 5000 km?

$$L_{Edd} = \frac{4\pi c G M}{\sigma_T/m_H}$$

remember

$$\boxed{L_{Edd} = \frac{4\pi c G M}{\kappa_T}}$$

$$\boxed{L_{Edd} \simeq 10^{38}\,\mathrm{erg/s}\,(M/M_\odot)}$$

$$\boxed{L = 4\pi R^2 \cdot \sigma T^4} \rightarrow \boxed{T = \left(\frac{L}{4\pi R^2 \sigma}\right)^{1/4}}$$

$\sigma = 10^{24}\,[\mathrm{erg/s/cm^2/keV^4}]$

$$T = \left(\frac{1.4 \times 10^{38}\,[\cancel{\mathrm{erg/s}}]}{4\pi \times (10 \times 10^5\,\cancel{\mathrm{cm}})^2 \cdot 10^{24}\,\cancel{\mathrm{erg/s/cm^2}}/\mathrm{keV^4}}\right)^{1/4} [\mathrm{keV}]$$

$$= (11.1)^{\frac{1}{4}}\,[\mathrm{keV}] \simeq 2\,\mathrm{keV}$$

$$L_{Edd} = \frac{4\pi c G M}{\kappa_T}\frac{M_\odot}{M_\odot}$$

$$= \frac{4\pi}{\kappa_T}\frac{c^3 G M_\odot}{c^2}\left(\frac{M}{M_\odot}\right)$$

$$= \frac{\cancel{4\pi}}{\kappa_T} \cdot \frac{2GM_\odot}{c^2} \cdot c^3 \left(\frac{M}{M_\odot}\right)$$

$$= \frac{2\pi}{0.4\,[\mathrm{cm^2/g}]} \cdot 3\,[\mathrm{km}]\,(3 \times 10^{10}\,[\mathrm{cm/s}])^3 \cdot \left(\frac{M}{M_\odot}\right)$$

$$T = \left(\frac{L}{4\pi R^2 \sigma}\right)^{1/4}$$

$$= \left(\frac{10^{38}\,\mathrm{erg/s}}{10 \cdot (5000\,\mathrm{km})^2\,10^{24}}\right)^{1/4}$$

$$\simeq 208\,[\mathrm{keV}] \simeq \boxed{80\,\mathrm{eV}}$$

historical

2 − 10 keV

$0.4\,[\mathrm{cm^2/g}]$

$(3\times10^{10})^3 (\mathrm{cm/s})^3 \cdot \left(\frac{M}{M_\odot}\right)$

$= \frac{2\pi}{0.4}\cdot 3\times10^5 \cdot (3\times10^{10})^3$

$\cdot \left[\frac{\mathrm{erg}}{s}\right]\left(\frac{M}{M_\odot}\right)$

$\approx 10^{\boxed{38}}\left(\frac{M}{M_\odot}\right)(\mathrm{erg/s})$

2 − 10 keV

soft X-ray → hard X-ray

4. **What are those sources, neutron stars or white dwarfs shining at the Eddington luminosity?**

Discovery of intense X-ray bursts from the globular cluster NGC6624

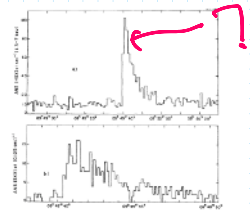


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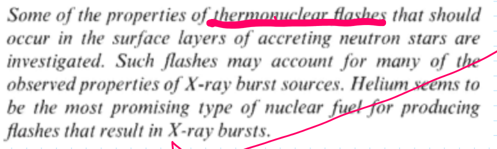
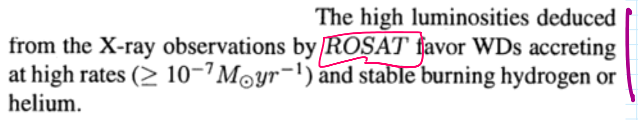
FIG. 2.—Burst of 09ʰ49ᵐ September 28. Upper histogram is intensity profile as seen by HXX with 1 s integration intervals. Lower histogram is the event as seen by SXX with 0.125 s integration intervals.

X-ray burst and neutron-star themonuclear fashes



Some of the properties of thermonuclear flashes that should occur in the surface layers of accreting neutron stars are investigated. Such flashes may account for many of the observed properties of X-ray burst sources. Helium seems to be the most promising type of nuclear fuel for producing flashes that result in X-ray bursts.

Super-soft X-ray sources in the fields of the Magellanic Clouds (by ROSAT)



The high luminosities deduced from the X-ray observations by *ROSAT* favor WDs accreting at high rates ($\geq 10^{-7} M_\odot yr^{-1}$) and stable burning hydrogen or helium.

5. **Are there "Super-Eddington sources" whose luminosities exceed the Eddington luminosity?**

Yes!!

An ultraluminous X-ray source powered by an accretion neutron star

Editorial Summary

What drives ultraluminous X-ray sources?

Ultraluminous X-ray sources (ULXs) are non-nuclear point sources that are widely believed to contain either ~~intermediate mass black holes~~ or smaller, stellar mass black holes accreting from a binary companion. The study of ULXs provides information about black hole formation and/or modes of high Eddington rate accretion.

IMBH

The pulsating source is spatially coincident with a variable source that can reach an X-ray luminosity in the 0.3–10 kiloelectronvolt range of 1.8 × 10^{40} ergs per second. This association implies a luminosity of about 100 times the Eddington limit for a 1.4-solar-mass object.

<https://www.nature.com/articles/nature13791?proof=t>