Very fundamental physics:

1. What are the **three** most basic physical parameters of the universe?

$c, G, \hbar$

$c = 2.9975\times 10^{8}\ [m/s]$ ← remember!! $3\times10^{8}\ [m/s]$

$G = 6.67\times 10^{-11}\ [N\cdot m^2/kg^2]\ [m^3/kg/s^2]$

$\hbar = \frac{h}{2\pi} = 1.05\times 10^{-34}\ [J\cdot s]\ [kg\ m^2/s]$

2. Indicate that the dimension of "length", "mass", "time" can be reproduced from these three parameters. What are the meaning of these values?

$\hbar c = [kg\cdot m^3/s^2] \quad \frac{\hbar c}{G} \left[\frac{\cancel{m^3}\cdot kg/\cancel{s^2}}{\cancel{m^3}/kg/\cancel{s^2}}\right] [kg^2]$

mass
length
time

$\text{Planck mass} = \sqrt{\frac{\hbar c}{G}}\ [kg] \approx 2\times10^{-8}\ [kg]$

minimum BH mass possible

$\text{Planck length} = \sqrt{\frac{G\hbar}{c^3}} \approx 1.6\times10^{-35}\ [m]$

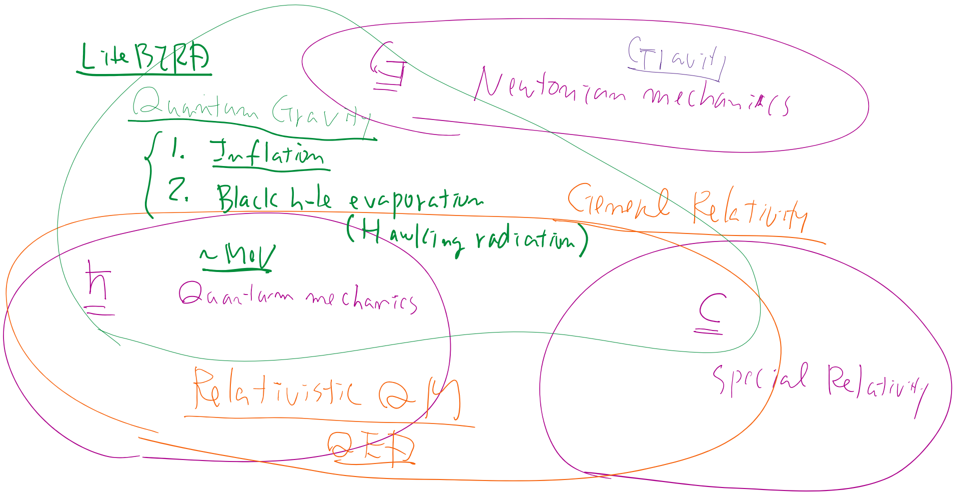
$\text{Planck time} = \frac{1}{c}\,\text{Planck length} = \sqrt{\frac{G\hbar}{c^5}} \approx 5\times10^{-44}\ [sec]$

$\boxed{R_s = \frac{2GM}{c^2}} \longrightarrow M \fallingdotseq \frac{c^2 R_s}{2G} = \frac{c^2}{G}\sqrt{\frac{G\hbar}{c^3}} = \sqrt{\frac{c^4 G\hbar}{G^2 c^3}} = \sqrt{\frac{c\hbar}{G}}$

Schwarzschild radius

3. Obtain the relationship between the Planck length and the Planck mass. Compare the Planck length with the Schwarzschild radiu s of a particle having the Planck mass.

4. In which physical theories, which of the three parameters appear? Are there physical theories in which all the three parameter s appear? What is the implication of this fact?



Electromagnetism:

1. Express Coulomb's law in the MKSA unit and Gauss unit.

$q\ [C] \quad [A] = [C/s]$

$F = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r^2}$

$F = \frac{1}{4\pi\mu_0}\frac{mm'}{r^2}$

$F = \frac{qq'}{r^2}$

$F = mm'$

$$F = \frac{1}{4\pi\mu_0}\frac{mm'}{r^2}$$

$$F = \frac{ee'}{r^2}, \quad F = \frac{mm'}{r^2}$$

2. What is the dimension of the electric/magnetic charge in the Gauss unit?

$$[g \cdot cm/s^2] = \frac{e^2}{[cm^2]} \qquad e^2 = [g \cdot cm^3 / s^2]$$

$$e,\ m = [g^{\frac{1}{2}} \cdot cm^{\frac{3}{2}} \cdot s^{-1}]$$

3. Express the light-velocity c using the vacuum permittivity ε_0 and the vacuum permeability μ_0 in MKSA.

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

4. Express Maxwell equations in the MKSA unit and the Gauss unit.

	MKSA	Gauss
$\mathrm{div}\,\mathbb{B} =$	0	0
$\mathrm{div}\,\mathbb{D} =$	ρ	$4\pi\rho$
$\mathrm{rot}\,\mathbb{H} =$	$j + \frac{\partial \mathbb{D}}{\partial t}$	$\frac{4\pi}{c} j + \frac{1}{c}\frac{\partial \mathbb{D}}{\partial t}$
$\mathrm{rot}\,\mathbb{E} =$	$-\frac{\partial \mathbb{B}}{\partial t}$	$-\frac{1}{c}\frac{\partial \mathbb{B}}{\partial t}$
$\mathbb{D} =$	$\varepsilon \mathbb{E}$	$\frac{\varepsilon}{\varepsilon_0}\mathbb{E}$
$\mathbb{B} =$	$\mu \mathbb{H}$	$\frac{\mu}{\mu_0}\mathbb{H}$

5. Indicate the relation between the magnetic energy density ε and the magnetic flux density B. Use both the Gauss unit, where ε is in [erg/cm^3] and B is in [gauss], and the MKSA unit, where ε is in [J/m^3] and B is in [T]. Examine that both agree.
   - A memo on the magnetic field density expressed in differnt units (pdf)

$$\varepsilon\,[erg/cm^3] = \frac{1}{8\pi}\left(B[\mathrm{Gauss}]\right)^2$$

$$\varepsilon'\,[J/m^3] = \frac{1}{2\mu_0}\left(B'(T)\right)^2 \qquad \mu_0 = 4\pi \times 10^{-7}\,[N/A^2]$$

$$[10^7 erg/(10^2 cm)^3] = 10\,[erg/cm^3] \longrightarrow \varepsilon' = \frac{1}{10}\varepsilon$$

$$1\,T \leftrightarrow 10^4\,(\mathrm{Gauss})$$

$$\varepsilon' = \frac{1}{2 \times 4\pi \times 10^{-7}}\left(10^{-4} B(\mathrm{gauss})\right)^2 = \frac{10^{-8}}{8\pi \cdot 10^{-7}} B^2 = \frac{1}{10}\varepsilon$$

Remember the following numbers/formula which often appear in physics. For numbers, a few significant digits are sufficient:

1. Light velocity

$$c \simeq 3 \times 10^{10}\ cm/s$$

2. Consider X-rays with the energy E [keV] and the wavelength λ [A]. Obtain the relationship between λ and E.

$$E = h\nu = \frac{hc}{\lambda} = \frac{\hbar c \cdot 2\pi}{\lambda} = \frac{1.973\,(keV \cdot Å) \cdot 2\pi}{\lambda} \qquad (c = \lambda\nu)$$

$$E \cdot \lambda = 1.973 \times 2\pi \times (keV \cdot Å] = 12.4\,[keV \cdot Å]$$

$$1\,Å \simeq 12.4\,keV, \qquad 1\,keV \simeq 12.4\,Å$$

3. Boltzmann constant k.

$$k = 1.38 \times 10^{-16}\,(erg \cdot K^{-1}]$$

4. 1 eV corresponds to approximately ** [K] or ** [erg].

$$1\,eV \simeq 1.6 \times 10^{-12}\ erg$$

$$E = kT \qquad T = \frac{E}{k}$$

$$T \simeq \frac{1.6 \times 10^{-12}\,erg}{1.38 \times 10^{-16}\,erg \cdot K^{-1}}$$

$$1\,eV \simeq 11604\,[K] \simeq 10^4\,[K]$$

$$e \quad \sim \frac{\quad}{1.38\times10^{-16}\,\mathrm{erg\cdot K^{-1}}}$$

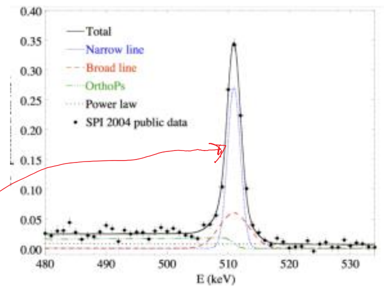
$$\boxed{1\,\mathrm{eV}} \approx 11604\,[\mathrm{K}] \approx 10^4[\mathrm{K}]$$

$1\,\mathrm{eV} \approx 10^4\,(\mathrm{K})$ observation

$1\,[\mathrm{keV}] \approx 10^7\,[\mathrm{K}] \Longleftrightarrow 10\,[\mathrm{keV}] \approx 10^8\,[\mathrm{K}]$

5. Electron rest mass (in keV).

$m_e c^2 = 511\,\mathrm{keV}$ $\qquad e^+ + e^- \rightleftarrows \gamma + \gamma$ (511 keV, 511 keV)



Electron-positoron annihillation line at 511 keV from the Galactic Center

6. Approximate nucleon (=proton or neutron) mass (in MeV or GeV). Which is heavier, proton or neutron?

$\boxed{m_p \approx m_n \approx 1\,\mathrm{GeV}}$ $\qquad m_n \approx 939.6\,\mathrm{MeV}/c^2$, $\quad m_p \approx 938.3\,\mathrm{MeV}/c^2$

β decay: $n \to p + e^- + \nu$

electron capture — inside of neutron star

7. Stephan-Boltzmann constant σ in [erg/s/cm²/keV⁴]

$\sigma T^4\,[\mathrm{erg/cm^2/s}]$ $\qquad \sigma \approx 10^{24}\,[\mathrm{erg/s/cm^2/keV^4}]$

1. Using above, what is the luminosity of the blackbody emitting neutron star at 2 keV with the radius 10 km?

X-ray burst

$L\,[\mathrm{erg/s}] = 4\pi R^2 \cdot \sigma T^4$

$= 4\pi \cdot (10\cdot10^5\,\mathrm{cm})^2 \cdot 10^{24}\,[\mathrm{erg/s/cm^2/keV^4}] \cdot (2\,\mathrm{keV})^4$

$\approx 2\times10^{38}\,[\mathrm{erg/s}]$

8. Fine structure constant (both formula and value).

$$\boxed{\frac{e^2}{\hbar c} = \frac{1}{137}}$$

9. ℏ c

$1973\,\mathrm{eV\cdot Å} \approx 2\,\mathrm{keV\cdot Å}$

10. Classical electron radius r_0 (formula and value)

$$\boxed{mc^2 = \frac{e^2}{r}}$$

$$r = \frac{e^2}{mc^2} = \frac{1}{mc^2}\left(\frac{e^2}{\hbar c}\right)\hbar c = \frac{1}{511\,\mathrm{keV}}\cdot\frac{1}{137}\cdot 2\,\mathrm{keV\cdot Å}$$

$$= \boxed{3\times10^{-5}\,\mathrm{Å}}$$

11. Express the Thomson scattering cross-section with r_0, and remember the value.

$$\sigma_T = \frac{8}{3}\pi r_0^2 \approx \frac{8}{3}\pi\times(3\times10^{-5}\times10^{-8}\,\mathrm{cm})^2 \approx 7.5\times10^{-25}\,(\mathrm{cm^2})$$

$$\boxed{\sigma_T = 6.65\times10^{-25}\,(\mathrm{cm^2})}$$

~100 km, space; artificial satellites

12. Bohr radius (both formula and number)

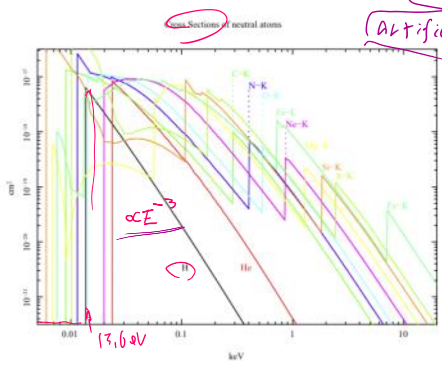
$$\boxed{m\frac{v^2}{r} = \frac{Ze^2}{r^2}, \quad mrv = \hbar}$$

$v = \frac{\hbar}{mr}$ $\qquad m\left(\frac{\hbar}{mr}\right)^2 = \frac{e^2}{r^2}$ $\qquad \frac{m\hbar^2}{m^2 r^2} = \frac{e^2}{r}$ $\qquad r = \frac{\hbar^2}{me^2}$

$mv^2 = \frac{e^2}{r}$

$$r = \frac{\hbar^2}{me^2 Z} = \frac{(\hbar c)^2}{me^2c^2} = \frac{1}{m}\cdot\frac{\hbar c}{e^2}\cdot\frac{\hbar c}{c^2} = \frac{\hbar c}{mc^2}\cdot\frac{\hbar c}{e^2}\,137$$

$$= \frac{1}{511\,\mathrm{keV}}\cdot 1973\,\mathrm{keV\cdot Å}\cdot 137 = \boxed{0.52\,\mathrm{Å}}$$



$$\frac{e^2}{\hbar c} = \frac{1}{137}$$

balloon, sounding rocket, upper atmosphere; hard X-rays ~50–100 keV

13. Lyman edge energy of hydrogen (both formula and number in eV)

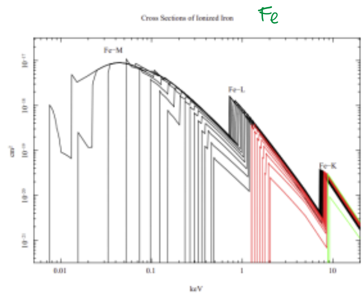
13.6 eV

$$E = \frac{1}{2}mv^2 - \frac{Ze^2}{r} = -\frac{1}{2}\frac{Ze^2}{r} = -\frac{1}{2}\frac{Ze^2}{\hbar^2/me^2} = -\frac{1}{2}\frac{me^4Z^2}{\hbar^2} = -\frac{1}{2}\cdot\frac{mc^2e^4}{(\hbar c)^2}Z^2$$

$$= -\frac{1}{2}\,mc^2\cdot\left(\frac{e^2}{\hbar c}\right)^2 = -\frac{1}{2}\cdot 511\cdot10^3\,\mathrm{eV}\cdot\left(\frac{1}{137}\right)^2 = 13.6\,[\mathrm{eV}]$$

$E\cdot\lambda = 12.4\,[\mathrm{keV\cdot Å}]$ $\qquad \lambda = \frac{12.4\,[\mathrm{keV\cdot Å}]}{E\,[\mathrm{keV}]}$

$\lambda = \frac{12.4\times10^3\,\mathrm{eV\cdot Å}}{13.6\,\mathrm{eV}} = 912\,[\mathrm{Å}]$



14. Lyman edge energy (in keV) of heavy atoms with the atomic number Z.

ion with Z $\to$ $\boxed{13.6\,\mathrm{eV}\cdot Z^2}$

Fe → Z = 26

Hydrogenic Iron ion, Z = 26

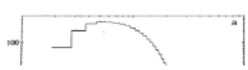
Lyman edge energy (in keV) of hydrogenic iron // Fe

$13.6\,\mathrm{eV}\cdot(26)^2 = \boxed{9.2\,\mathrm{keV}}$

15. The earth atmosphere is "opaque" to soft (<<10 keV) X-rays. Which elements will most affect the X-ray absorption in the atmosphere?

C, N, O

photoelectric absorption by.

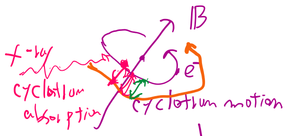


C, N, O

photoelectric absorption by

$13.6\,\mathrm{eV}\cdot(26)^2 = \boxed{9.2\,\mathrm{keV}}$

16. Using the value of Bohr magneton ℏ e/(2mec) = 9.3e-21 [erg/gauss], derive the relation between the energy of cyclotron absorption line [keV] and B [gauss].



cyclotron emission

$\frac{\hbar e}{2mc} = 9.3\times10^{-21}$ [erg/gauss]

m is electron mass

$\frac{m v^2}{r} = \frac{e v B}{c m}$

$\omega = v/r = \frac{eB}{mc}$

cyclotron frequency

$\boxed{1\,\mathrm{eV} \approx 1.6\times10^{-12}\,\mathrm{erg} \quad 1\,\mathrm{keV} \approx 1.6\times10^{-9}\,\mathrm{erg}}$

$E = \hbar\omega$ Landau level

$= \frac{2\hbar eB}{2mc} = \frac{\hbar e}{2mc}\cdot 2B$

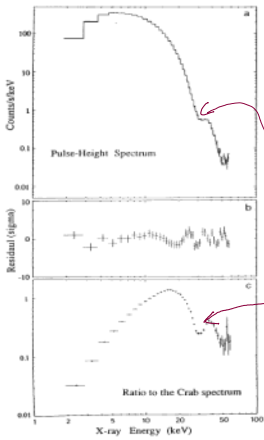
$= 9.3\times10^{-21}\,(\mathrm{erg/gauss})\cdot 2B$

$= \frac{9.3\times10^{-21}}{1.6\times10^{-9}}\,(\mathrm{keV/gauss})\,2B$

$\fallingdotseq 5.8\times10^{-12}\,(\mathrm{keV/G})\cdot 2\left(\frac{B}{10^{12}\,G}\right)10^{12}\,G$

$= 11.6\,\mathrm{keV}\left(\frac{B}{10^{12}\,G}\right)$

$$E \fallingdotseq 12\,\mathrm{keV}\left(\frac{B}{10^{12}\,G}\right)$$



from X0331+53 Cyclotron absorption line detected by the Ginga satellite

cyclotron absorption line @28.5 keV

⇓

$B \approx 2.5\times10^{12}\,G$