Lesson 11 X-ray emission mechanism (1)

2021年6月15日

1. Optical Depth

1. Explain concept of the optical depth

I_ν [erg/s/cm²/Hz/str] specific intensity

$Ie^{-\tau}$, $I_\nu e^{-\tau_\nu}$

$\tau \gg 1$: $Ie^{-\tau} \to 0$ opaque
$\tau \ll 1$: $I_\nu (1-\tau_\nu)$ transparent

2. Express the optical depth with several different combinations of physical parameters

$\tau = \alpha\,[\mathrm{cm^{-1}}]\, L\,[\mathrm{cm}]$
$= \kappa\,[\mathrm{cm^2/g}] \cdot \rho\,[\mathrm{g/cm^3}] \cdot L\,[\mathrm{cm}]$
$= N_H\,[\mathrm{cm^{-2}}] \cdot \sigma_H\,[\mathrm{cm^2}]$

α: absorption coefficient
κ: opacity, ρ: mass density
N_H: hydrogen column density, σ_H: cross-section per hydrogen

a photon goes without being absorbed / being scattered — probability that: $e^{-\tau}$

$\langle \tau \rangle = \int_0^\infty \tau\, e^{-\tau} d\tau = 1$

Average optical depth of a photon not being absorbed or scattered

Stellar atmosphere ($\tau = 0$, $\tau = 1$)

l: mean free path [cm]
average photon path-length without being absorbed or scattered

$\langle \tau \rangle = \alpha\, l = 1$, $\alpha l = 1$, $l = \frac{1}{\alpha}$

3. When photons interact with matter, what will happen? State the **two** fundamental processes.

- absorption → photons disappear
- scattering → photons does not disappear

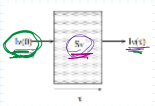
τ_a: absorption optical depth
τ_s: scattering optical depth

$$B_\nu(T) = \frac{2}{c^2}\frac{h\nu^3}{e^{h\nu/kT}-1} \quad [\mathrm{ergs/s/cm^2/Hz/str}]$$

Planck function

2. Radiative Transfer

1. Formulate the simplest **radiative transfer equation** in the uniform medium, where the **specific intensity** is Iν [erg/s/cm²/Hz/str] and the **source function** is Sν [erg/s/cm²/Hz/str], and solve it.



$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

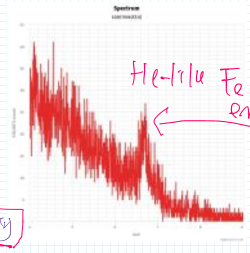
$\int \frac{dI_\nu}{S_\nu - I_\nu} = \int d\tau_\nu$, $-\ln(S_\nu - I_\nu) = \tau_\nu + C$

$S_\nu - I_\nu = Ae^{-\tau_\nu}$, $I_\nu = S_\nu + Ae^{-\tau_\nu} = S_\nu + (I_\nu(0) - S_\nu)e^{-\tau_\nu}$

$\tau = 0 \to I_\nu(0) = S_\nu + A$, $A = I_\nu(0) - S_\nu$

$= I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$

https://darts.isas.jaxa.jp/astro/udon2/plot.html?SAT=SUZAKU&OBS=402070010&REGION=%27circle(748,766,45)%27&VER=3.0



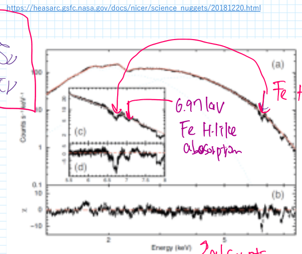
He-like Fe emission line @ 6.67 keV

Cir X-1 when dim (by Suzaku)

2. Explain the physical meaning of the radiative transfer equation *intuitively*.

When $S_\nu > I_\nu$, $\frac{dI_\nu}{d\tau_\nu} > 0$: I_ν increases with τ_ν
When $S_\nu < I_\nu$, $dI_\nu/d\tau_\nu < 0$: I_ν decreases with τ_ν
→ I_ν approaches S_ν with τ_ν

https://heasarc.gsfc.nasa.gov/docs/nicer/science_nuggets/20181220.html



Fe He-like absorption @ 6.67 keV; Fe H-like absorption 6.97 keV; absorption lines

Cir X-1 when bright (by NICER)

N.S; ionized: He-like, H-like

3. What will be the solutions of the radiative transfer equation for the optically *thick* limit and the *thin* limit?

$\tau_\nu \gg 1$: $I_\nu \simeq S_\nu$
$\tau_\nu \ll 1$: $I_\nu \simeq I_\nu(0)(1-\tau_\nu) + \tau_\nu S_\nu$

4. Explain the difference of the **thermal emission** and the **blackbody emission**

$S_\nu = B_\nu$; $I_\nu \simeq B_\nu$
$\tau_\nu \gg 1$ ⇓ $I_\nu \simeq S_\nu \simeq B_\nu$

5. In the optically thin limit, consider the two cases when there is a strong input emission into the medium, and there is no input emission. What do you observe in these two cases?

$I_\nu \simeq I_\nu(0)(1-\tau_\nu)$; $I_\nu \simeq \tau_\nu S_\nu$

6. Indicate that intensity of any thermal emission does NOT exceed that of the blackbody emission.

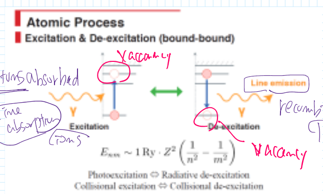
$I_\nu = I_\nu(0)e^{-\tau_\nu} + B_\nu(1-e^{-\tau_\nu}) \le B_\nu$

τ_ν 多: $I_\nu \simeq B_\nu$; $\tau_\nu = 0$: $I_\nu = I_\nu(0) \le B_\nu$

3. Absorption lines and emission lines

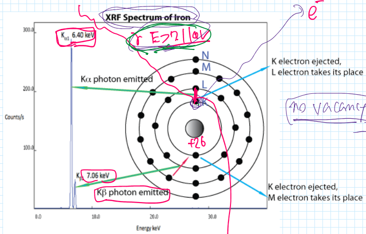
1. Explain the process of producing absorption lines and emission lines

https://github.com/tenoto/repository/blob/master/docs/AHCookBook_v1.02.pdf



Photons absorbed, vacancy — line absorption (ions); recombination line; vacancy

http://www.xrfresearch.com/x-ray-fluorescence/



e^-; $\Delta E = 6.4$ keV; no vacancy

2. Explain the two different mechanisms of producing emission lines

emission:
- recombination line → highly ionized ions
- fluorescence line → neutral or low ionized ions

3. What are the energies of the Fe-K emission lines for neutral iron atom (fluorescent line), He-like iron ion, and the H-like iron ion?

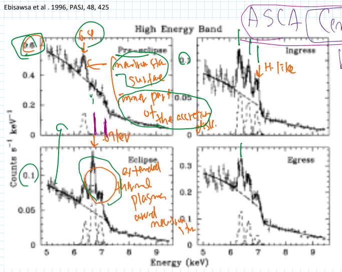
Neutral 6.4 keV
He-like 6.67 keV ← triplet
H-like 6.97 keV ← Kα + Kβ

Not resolved by CCD; with XRISM resolved

4. Show examples of the observations where these three Fe emission lines are detected simultaneously.

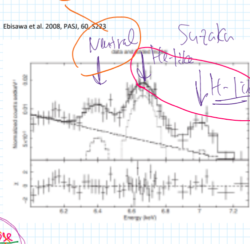
Many!!

Ebisawa et al. 1996, PASJ, 48, 425



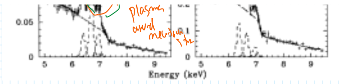
ASCA Cen X-3 binary X-ray pulsar; eclipse

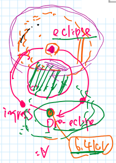
Ebisawa et al. 2008, PASJ, 60, S223



Suzaku Galactic Ridge; Neutral, He-like, H-like; cold material; hot plasma

plasma
Fe
Fe





4. How to express X-ray energy spectra?

1. Explain different ways of expressing the energy spectra?

https://www.isas.jaxa.jp/home/ebisawalab/ebisawa/TEACHING/2016Note.pdf, page 27

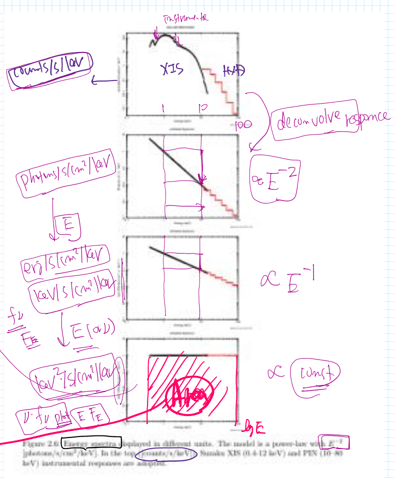


Figure 2.6: Energy spectra displayed in different units. The model is a power-law with E^{-2} [photons/s/cm²/keV]. In the top, counts/s/keV; Suzaku XIS (0.4-12 keV) and PIN (10-80 keV) instrumental responses are adopted.

2. Often " ν f_ν plot" (= E F_E plot) is adopted to express energy spectra. What is this? What is the merit of using ν f_ν plot?

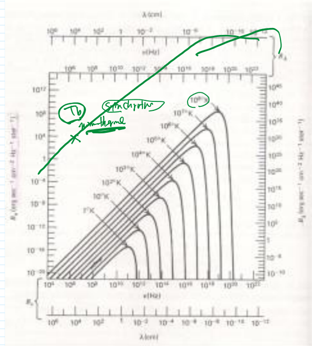
f_ν [erg/s/cm²/keV]

$$F(x) = \int f_\nu \, d\nu \quad [\mathrm{erg/s/cm^2}] = \int \nu f_\nu \frac{d\nu}{\nu} = \int \nu f_\nu \, d(\ln\nu) \propto \int \nu f_\nu \, d(\log \nu)$$

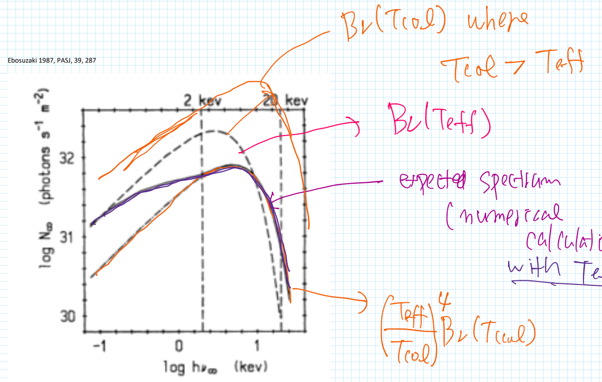
5. Blackbody Emission

1. Derive the formula of the Planck function for the low-energy limit (Rayleigh-Jeans law) and high-energy limit (Wien function)

Rybicki and Lightman "Radiative Processes in Astrophysics"



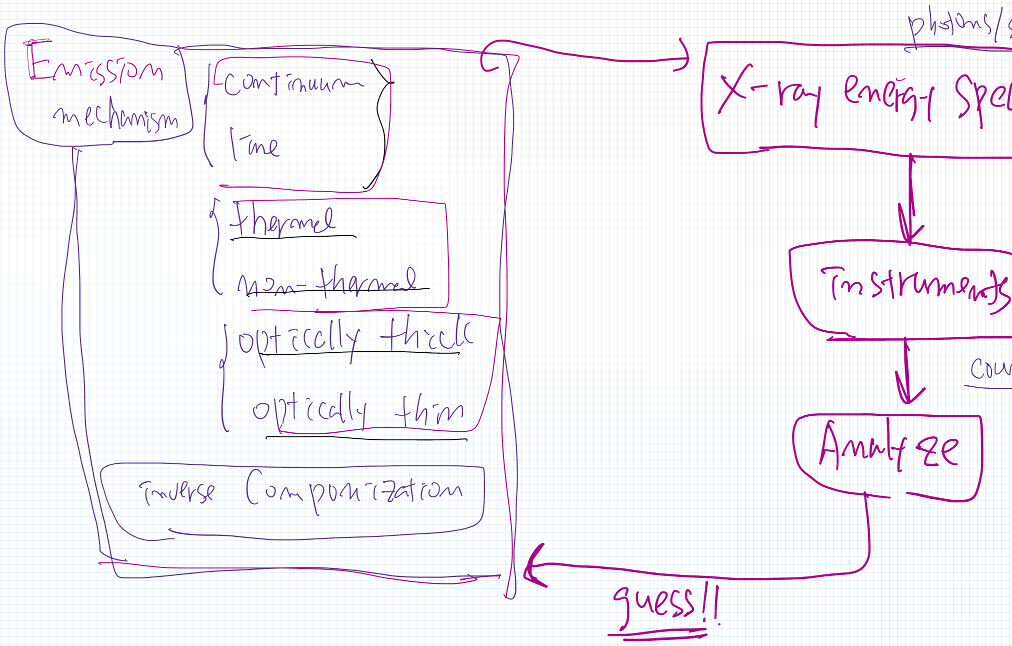
Ebisuzaki 1987, PASJ, 39, 287



2. Obtain the energy at which the Planck function peaks.

3. Derive the Stefan-Boltzman constant from the Planck function

4. Explain the difference of the effective temperature, color temperature, and brightness temperature



5. Draw the black body spectrum with the temperature E=kT in a wide energy range using the ν f_ν plot. Observe the energy spectral distribution around kT (widely distributed, or narrowly distributed).
Are there any continuum spectra which are more concentrated (narrowly distributed) around kT?

on)

ff

s/cm^2/keV

ctk

nts/ channel