20230525 University of Tokyo

2023年5月25日　14:35

Black holes

Ken EBISAWA (ISAS/JAXA)

Today, I will use "One Note", and I will write on the white-board by hand,
Please take note by hand, as much as possible!
Also, bring a calculator, so that we can carry out simple calculations during the lecture togetyer.

This One Note and PDF output will be made available on my HP,
https://www.isas.jaxa.jp/home/ebisawalab/ebisawa/
If you have questions, please write on chat, or speak-up!

Very fundamental physics:

1. What are the **three** most basic physical parameters of the universe?

c, G, ħ

$c = 2.99 \times 10^8$ [m/s] ~ 3×10^8 [m/s]

$G = 6.67 \times 10^{-11}$ [N/m²/kg²] [m³/kg/s²]

$\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34}$ [J·s] [kg·m²/s]

2. Indicate that the dimension of "length", "mass", "time" can be reproduced from these three parameters. What are the meaning of these values?

$\hbar c$ [kg·m³/s²]

$\frac{\hbar c}{G}$ $\left[\frac{kg \cdot m^3/s^2}{m^3/kg/s^2}\right]$ [kg²]

$\sqrt{\frac{\hbar c}{G}}$ [kg] ~ 2×10^{-8} [kg] ← Planck mass ← the smallest BH in the Universe

Planck length $\doteq \sqrt{\frac{G\hbar}{c^3}} \approx 1.6 \times 10^{-35}$ [m] ← minimum length

Planck time $= \frac{1}{c}$ Planklength $\approx 5 \times 10^{-44}$ [sec] ← Shortest time

Schwarzschild radius $R_s = \frac{2GM}{c^2}$ → $M \doteq \frac{c^2 R_s}{G} = \frac{c^2}{G}\sqrt{\frac{G\hbar}{c^3}} = \sqrt{\frac{\hbar c}{G}}$

for the object with mass M

3. Obtain the relationship between the Planck length and the Planck mass. Compare the Planck length with the Schwarzschild radius of a particle having the Planck mass.

Quantum Gravity — not established yet!!

4. In which physical theories, which of the three parameters appear? Are there physical theories in which all the three parameters appear? What is the implication of this fact?

G: Newtonian mechanics

ħ: Quantum mechanics

c: Special Relativity

General Relativity

Relativistic quantum mechanics

2. What kinds of black holes exist?

Stellar-mass black holes

Super-massive black holes (SMBH)

Intermediate-Mass black holes?

Eddington limit

Primordial black holes??

3. Observations of black holes

"Direct" observations by gravitational wave

Measurements of mass

Observations by electromagnetic waves (This is astronomy!)

4. Future space astronomy projects

What to know about black holes

2021年4月20日 16:42

Basic of Black Holes:

1. **In Newtonian mechanics, obtain the radius of a star with mass M, where the escape-velocity is equal to the light velocity. Compare this with the Schwarzschild radius.**

$E = \frac{1}{2}mv^2 - G\frac{Mm}{r} = 0$ $\frac{1}{2}v_{esc}^2 - \frac{GM}{r} = 0$

$v_{esc}^2 = \frac{2GM}{r}$

$r = \frac{2GM}{v_{esc}^2}$ $v_{esc} \to c$ $\frac{2GM}{c^2} \to R_s$

2. **Mass of the black hole in the center of our Galaxy (Sgr A*)? (A latest result, Stellar Orbits around the black hole, Nobel Prize 2020!)**

$\sim 4\times10^6 M_\odot$ ← Super-massive Black Holes

Stellar mass black holes $\sim 3M_\odot - \sim 60M_\odot$

3. **Are there "intermediate mass black holes (IMBHs)" with a mass of 100-1000 Msolar? Which sources are the candidates of the intermediate mass black holes?**

origin still debated

Ultraluminous X-ray Sources ← ULXs { IMBH? / Stellar mass BH? (massive)

A 400-solar-mass black hole in the galaxy M82

Schwarzschild radius of Sun ~ 3km $R_s = 3\left(\frac{M}{M_\odot}\right)$ km

$1° = 60' = 3600''$
$1' = 60''$

Feel the "size" of Black Holes:

1. **Estimate apparent size of a black hole, dividing the Schwarzschild radius by the distance to the source. Which is easier to "resolve", stellar-mass black hole in our Galaxy, or super-massive black holes in other galaxies?**

$\Delta\theta \approx \frac{R_s}{d} = \frac{2GM/c^2}{d} = \frac{3\,km\; M/M_\odot}{d} = \frac{30\,km\,(M/10M_\odot)}{(d/10kpc)\cdot 10kpc} \approx \frac{30km}{10kpc}\left(\frac{M/10M_\odot}{d/10kpc}\right)$

$\approx 10^{-16}[rad]\left(\frac{M/10M_\odot}{d/10kpc}\right)$

$\approx 2\times10^{-11}[arcsec]\left(\frac{M/10M_\odot}{d/10kpc}\right)$ ← stellar mass BH — not possible to resolve

(1pc ≈ 3×10^18 [cm]) parsec

Sgr A* { $M \approx 4\times10^6 M_\odot$, $d \approx 8$ kpc

$\Delta\theta \approx 2\times10^{-11}[arcsec]\,\frac{4\times10^5}{0.8} \approx 10^{-5}[arcsec]$

$\approx 10\,[\mu arcsec]$ ← possible to resolve

2. **Estimate spatial resolution of a radio interferometer at 1mm, where the base-line is 10,000 km (=maximum on Earth). Also, estimate spatial resolution of an X-ray interferometer at 1 A, with the base-line 10 m. Which has better spatial resolution?**

$\Delta\theta \approx \frac{\lambda}{D} \approx \frac{1mm}{10^4 km} \approx \frac{10^{-3}m}{10^7 m} \approx 10^{-10}[rad]$

$\approx \frac{10^{-10}}{\pi}\times180\times60\times60\,[arcsec] = 2\times10^{-5}[arcsec]$

$\approx 20\,[\mu arcsec]$

VLBI

Event horizon telescope

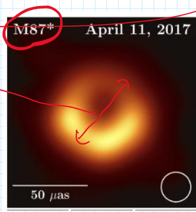
HALCA - The first Space LBI project ← Space VLBI

Micro-arcsecond X-ray Imaging Mission (MAXIM)

$\Delta\theta \approx \frac{1A}{10m} \approx \frac{10^{-10}m}{10m} \approx 10^{-11}[rad]$

"X-ray ..."

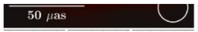
3. **According to General Relativity, "Photon-capture radius" of a black hole is √27 r_g, where r_g is the gravitational radius (=GM/c²). In 2019, the Event Horizon Telescope observed a bright photon "ring" around central blackhole M87 (distance = 16.8 Mpc), where the diameter was 42 uas. By identifyng the ring radius as the photon-capture radius, estimate the black hole mass. (see 2019ApJ...875L...1E)**



$\Delta\theta = 42\,\mu as$

$= \frac{2\times\sqrt{27}\,\frac{GM}{c^2}\frac{M_\odot}{M_\odot}}{16.8\,Mpc}$

$\frac{42\times10^{-6}}{2\times10^5} = 2\sqrt{27}\;1.5\,km\,(M/M_\odot)$



$$\frac{42\times10^{-6}}{2\times10^{5}} = \frac{2\sqrt{27}\ \overset{1.5}{3\text{km}}\ (M/M_\odot)}{16.8\times10^{6}\times3\times10^{18}[\text{km}]}$$

$$20\times10^{-11} = \frac{2\times 5.2\times1.5\times10^{3}}{5\times10^{23}}(M/M_\odot)$$

$$= 3\times10^{-20}(M/M_\odot)$$

$$\frac{M}{M_\odot} = \frac{2\times10^{-10}}{3\times10^{20}} \approx \frac{2}{3}\times10^{10} \approx 7\times10^{9}M_\odot$$

$$1\,\text{rad} = \frac{180^\circ}{\pi} = \frac{180\times3600}{\pi}\,\text{arcsec} = 2\times10^{5}[\text{arcsec}]$$

"X-ray interferometer"

4. **Assume that black hole is a sphere having the Schwarzschild radius (ignore General relativistic effects). Estimate "density" of the black hole, simply dividing the mass by the volume. Is the density smaller or larger for more massive black holes?**

R_s, M

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R_s^3} \approx \frac{1}{4}\frac{M}{R_s^3} = \frac{1}{4}\frac{M}{\left(\frac{2GM}{c^2}\right)^3} = \frac{1}{4}\frac{(M/M_\odot)M_\odot}{\left(3\text{km}\frac{M}{M_\odot}\right)^3}$$

$$= \frac{1}{4}\cdot\frac{M_\odot}{(3\times10^{5}\text{cm})^3}\left(\frac{M}{M_\odot}\right)^{-2}$$

$$= \frac{1}{4}\frac{2\times10^{33}[\text{g}]}{27\times10^{15}[\text{cm}^3]}\left(\frac{M}{M_\odot}\right)^{-2}$$

$$\approx \frac{2}{100}\times10^{18}\,(\text{g/cm}^3)\cdot\left(\frac{M}{M_\odot}\right)^{-2}$$

$$\approx 2\times10^{16}\left(\frac{M}{M_\odot}\right)^{-2}[\text{g/cm}^3]$$

$M\nearrow$ $\rho\searrow$

5. **Can the black hole "density" smaller than that of water? If yes, when?**

$1\,[\text{g/cm}^3]$

$$2\times10^{16}\left(\frac{M}{M_\odot}\right)^{-2} \ll 1$$

$$2\times10^{16} \ll \left(\frac{M}{M_\odot}\right)^{2}$$

$$\frac{M}{M_\odot} \gg (2\times10^{16})^{\frac{1}{2}} \approx 10^{8}$$

$$M \gg 10^{8}M_\odot$$

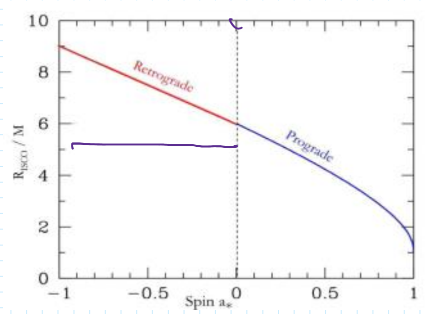
What more to know about black holes

2021年4月20日　16:42

Gravitational energy release from black holes

1. **Innermost Stable Circulr Orbits (ISCO) of non-rotating blackhole (Schwarzschild black hole)? What about a spinning black hole at the maximum rate (extreme Kerr black hole)?**

figure from 10.1007/s11433-018-9297-0



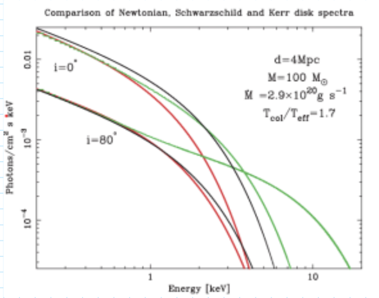
2. **Ignoring general relativity, estimate total energy E (potential energy + kinetic energy) of a mass m rotating at the ISCO around a black hole with mass M. Consider the Schwartzshild case and the extreme-Kerr case.**

3. **When the mass m reaches the ISCO from infinisty (where the initial velocity is assumed to be zero), the energy -E is released. What will be the energy conversion efficiency, η, where $-E = \eta mc^2$? Compare with the precise values using general relativity.**

4. **Compare with the efficiency of nuclear burning.**

Black hole spin

1. **How can we estimate black hole spin from X-ray observation?**



Ebisawa et al. (2003)

Measuring the spins of accreting black hole

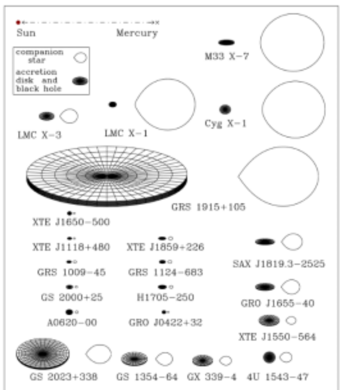


Table 1. Spin results to date for eight black holes[a].

	Source	Spin a_*	Reference
1	GRS 1915+105	>0.98	McClintock *et al* 2006
2	LMC X-1	$0.92^{+0.05}_{-0.07}$	Gou *et al* 2009
4	M33 X-7	0.84 ± 0.05	Liu *et al* 2008, 2010
3	4U 1543−47	0.80 ± 0.05	Shafee *et al* 2006
5	GRO J1655−40	0.70 ± 0.05	Shafee *et al* 2006
6	XTE J1550−564	$0.34^{+0.20}_{-0.28}$	Steiner *et al* 2010b
7	LMC X-3	<0.3[b]	Davis *et al* 2006
8	A0620−00	0.12 ± 0.18	Gou *et al* 2010

[a] Errors are quoted at the 68% level of confidence.
[b] Provisional result pending improved measurements of *M* and *i*.

Gravitational Wave and black holes

1. **What happens if two "bare" black holes (i.e., no accretion disks) merge in the *Binary Black Hole Coalescence*?**

Gravitational waves are emitted

→ No electromagnetic wave!!

Nobel prize 2017

2. **Let's assume that we detected a gravitational-event due to a blackhole merger, where amplitude of the gravitational wave is 10^{-21}. How much the distance between Sun and Earth (1 astronomical unit) varies due to this gravitational event? Answer with the unit of Bohr radius.**

↳ 500 light-sec = 3×10^{10} cm/s × 500 sec

= 1500×10^{10} cm = 1.5×10^{13} cm

$10^{-21}\times1.5\times10^{13}$ (cm) = 1.5×10^{-8} (cm)

≈ 3 times Bohr radius

↳ 0.5 Å

= 0.5×10^{-8} (cm)

3. **What about in the case of *neutron star mergers*?**

The historical first neutron star merger paper

These observations support the hypothesis that GW170817 was produced by the merger of two neutron stars in NGC 4993 followed by a short gamma-ray burst (GRB 170817A) and a kilonova/macronova powered by the radioactive decay of *r*-process nuclei synthesized in the ejecta.

Binary Neutron Star Mergers as the Production Site of Gold, Platinum, and Rare Earth Elements

4. **What do you expect in the case of the *binary super-massive black hole merger*, where both of the super-massive black holes are X-ray active AGN (i.e., they have accretion disks)?**

"A unique experiment to explore black holes" by ESA

See also the [You tube video by ESA]

Hawking radiation and black hole evaporation

1. **According to Hawking (1974), a black hole with the mass M has a temperature,**

$$T = \frac{\hbar c^3}{8\pi k G M},$$

→ $6 \times 10^{-8} \left(\frac{M_\odot}{M}\right)$ [K] M↗ T↘

Estimate the temperature in [K] for stellar mass black holes. Would it be possible to detect the "Hawking ratiation" from these black holes?

2. **Black holes may evaporate at a timescale of**

$$\tau \approx 400 \left(\frac{M}{10^{10}\ \mathrm{g}}\right)^3 \mathrm{s}.$$

Estimate the black hole mass which would evaporate in the age of the Universe

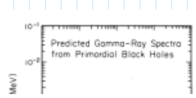
M↘ τ↘

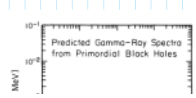
13.8 billion year

$M \approx 10^{15}$ [g] $\approx 5 \times 10^{-19} M_\odot$

3. **Estimate the temperature (in eV) of those "primordial black holes" which may have been created in the early universe and would evaporate in the age of the universe.**

4. **How can we search for such primordial black holes?**





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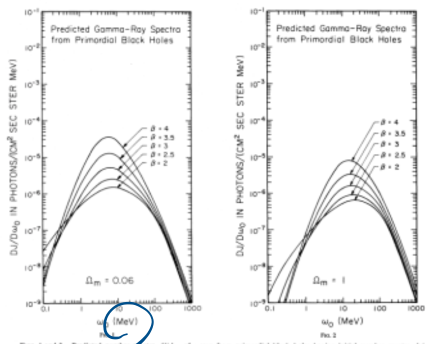

FIGS. 1 and 2.—Predicted number spectra $dJ/d\omega_0$ of γ-rays from primordial black holes having initial number spectra $dn/d(M_i/M_*) = 10^8$ pc$^{-3}(M_i/M_*)^{-\beta}$ for initial masses M_i around $M_* \simeq 5 \times 10^{14}$ g, where **β** is given values from 2 (*bottom curve*) to 4 (*top curve*) in steps of ½. Fig. 1 assumes the present matter density is 0.06 of the critical value for closure of the universe; Fig. 2 assumes it is at the critical value.

[Gamma-rays from primordial black holes]

$E = mc^2$

→ MeV

[Search for Gamma-ray emission from local primordial black holes with the Fermi Large Area Telescope]

Black holes

Ken EBISAWA (ISAS/JAXA)

1. What are the black holes?
2. What kinds of black holes exist?
3. Observations of black holes
4. Future space astronomy projects

1. What are the black holes

Three most basic physical parameters of the universe?

Planck length, Planck time, Planck mass

What is the meaning of the Planck mass?

General relativity --> Einstein equation --> Schwarzschild radius

Remember the Schwarzschild radius of the Sun and Earth!

Intuitive meaning of the Schwarzschild radius

Escape velocity (newtonian approximation)

$$E=\frac{1}{2}mv^2-G\frac{\ }{r}$$

Feel the "size" of Black Holes:

a. **Estimate spatial resolution of an radio interferometer at 1mm, where the base-line is 10,000 km (=maximum on Earth). Also, estimate spatial resolution of an X-ray interferometer at 1 A, with the base-line is 10 m. Which has better spatial resolution?**

Event horizon telescope

HALCA - The first Space VLBI project

Micro-arcsecond X-ray Imaging Mission

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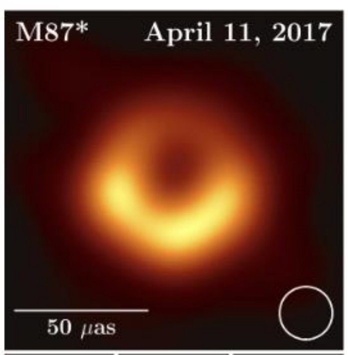

2. **Assume that black hole is a sphere having the Schwarzschild radius (ignore General relativistic effects). Estimate "density" of the black hole, simply dividing the mass by the volume. Is the density smaller or larger for more massive black holes?**

3. **Can the black hole "density" smaller than that of water? If yes, when?**

4. **Estimate light-crossing time of the Schwarzschild radius of a black hole with mass M.**

5. **Which is technically easier to detect X-ray variability taking place near the Schwarzschild radius, stellar mass black holes or super-massive black hole?**

2. What kinds of black holes exist?

Stellar-mass black holes

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