

$$In[14]:= \begin{pmatrix} \cos[\phi] & -\sin[\phi] & 0 \\ \sin[\phi] & \cos[\phi] & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[\theta] & 0 & \sin[\theta] \\ 0 & 1 & 0 \\ -\sin[\theta] & 0 & \cos[\theta] \end{pmatrix} \cdot \begin{pmatrix} \cos[\psi] & -\sin[\psi] & 0 \\ \sin[\psi] & \cos[\psi] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Out[14]= \{ \{ \cos[\theta] \cos[\phi] \cos[\psi] - \sin[\phi] \sin[\psi], \\ -\cos[\psi] \sin[\phi] - \cos[\theta] \cos[\phi] \sin[\psi], \cos[\phi] \sin[\theta] \}, \\ \{ \cos[\theta] \cos[\psi] \sin[\phi] + \cos[\phi] \sin[\psi], \cos[\phi] \cos[\psi] - \cos[\theta] \sin[\phi] \sin[\psi], \\ \sin[\theta] \sin[\phi] \}, \{ -\cos[\psi] \sin[\theta], \sin[\theta] \sin[\psi], \cos[\theta] \} \}$$

$$In[23]:= \text{Det} \left[\begin{pmatrix} \cos[\phi] & -\sin[\phi] & 0 \\ \sin[\phi] & \cos[\phi] & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[\theta] & 0 & \sin[\theta] \\ 0 & 1 & 0 \\ -\sin[\theta] & 0 & \cos[\theta] \end{pmatrix} \cdot \begin{pmatrix} \cos[\psi] & -\sin[\psi] & 0 \\ \sin[\psi] & \cos[\psi] & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

$$Out[23]= \cos[\theta]^2 \cos[\phi]^2 \cos[\psi]^2 + \cos[\phi]^2 \cos[\psi]^2 \sin[\theta]^2 + \\ \cos[\theta]^2 \cos[\psi]^2 \sin[\phi]^2 + \cos[\psi]^2 \sin[\theta]^2 \sin[\phi]^2 + \cos[\theta]^2 \cos[\phi]^2 \sin[\psi]^2 + \\ \cos[\phi]^2 \sin[\theta]^2 \sin[\psi]^2 + \cos[\theta]^2 \sin[\phi]^2 \sin[\psi]^2 + \sin[\theta]^2 \sin[\phi]^2 \sin[\psi]^2$$

$$In[24]:= \text{Simplify}[\cos[\theta]^2 \cos[\phi]^2 \cos[\psi]^2 + \cos[\phi]^2 \cos[\psi]^2 \sin[\theta]^2 + \\ \cos[\theta]^2 \cos[\psi]^2 \sin[\phi]^2 + \cos[\psi]^2 \sin[\theta]^2 \sin[\phi]^2 + \cos[\theta]^2 \cos[\phi]^2 \sin[\psi]^2 + \\ \cos[\phi]^2 \sin[\theta]^2 \sin[\psi]^2 + \cos[\theta]^2 \sin[\phi]^2 \sin[\psi]^2 + \sin[\theta]^2 \sin[\phi]^2 \sin[\psi]^2]$$

$$Out[24]= 1$$

$$In[26]:= \text{Simplify}[\\ \text{Dot}[\{ \cos[\theta] \cos[\phi] \cos[\psi] - \sin[\phi] \sin[\psi], -\cos[\psi] \sin[\phi] - \cos[\theta] \cos[\phi] \sin[\psi], \\ \cos[\phi] \sin[\theta] \}, \{ \cos[\theta] \cos[\psi] \sin[\phi] + \cos[\phi] \sin[\psi], \\ -\cos[\psi] \sin[\theta] - \cos[\theta] \cos[\phi] \sin[\psi], \cos[\phi] \sin[\theta] \}]]$$

$$Out[26]= 1$$

$$In[27]:= \text{Simplify}[\\ \text{Dot}[\{ \cos[\theta] \cos[\phi] \cos[\psi] - \sin[\phi] \sin[\psi], -\cos[\psi] \sin[\phi] - \cos[\theta] \cos[\phi] \sin[\psi], \\ \cos[\phi] \sin[\theta] \}, \{ \cos[\theta] \cos[\psi] \sin[\phi] + \cos[\phi] \sin[\psi], \\ \cos[\phi] \cos[\psi] - \cos[\theta] \sin[\phi] \sin[\psi], \sin[\theta] \sin[\phi] \}]]$$

$$Out[27]= 0$$