Type Ia supernovae (SNe Ia) are among the most luminous and energetic phenomena in the universe. They play a crucial role in cosmology as reliable distance indicators ("standard candles") and in the chemical evolution of the universe by synthesizing heavy elements, including iron-group elements such as chromium, manganese, iron, and nickel. While the progenitors of SNe Ia are widely believed to be white dwarfs (WDs) in binary systems, many fundamental questions remain unanswered. Additionally, the accumulation of observational data has revealed significant diversity among SNe Ia, which poses a serious challenge to their reliability as standard candles. To date, the two questions have been the focus of intense research for decades: "Whether the companion star is a non-degenerate star (e.g., a main-sequence star) or another WD?" and "Whether the progenitor mass is close to the Chandrasekhar mass ($\sim 1.4\ M_\odot$, referred as the near-$M_{ch}$ mass) or not ($\sim 1.0\ M_\odot$, referred as the sub-$M_{ch}$ mass)?" Together, these are referred to as the "progenitor problem of SNe Ia," recognized as one of the most significant open questions in modern astrophysics.

The X-ray observations of supernova remnants (SNRs) offer a unique opportunity to investigate the progenitors of SNe Ia. Although the sample size of SNe Ia remnants currently observable in X-rays is relatively small ($\approx 20$), thanks to the proximity of the remnants, their progenitor properties can be constrained for individual objects. In particular, measurements of the mass ratios of iron-group elements synthesized during the supernova explosion provide critical insights into the physical properties (e.g., mass, central density, metallicity, and neutronization degree) of the progenitor WD, making X-ray studies an invaluable resource for advancing our understanding of both stellar evolution and the explosion mechanisms of SNe Ia. However, interpreting the observed X-ray spectra of SNRs is often challenging due to the nonequilibrium states of ionization and temperature of the X-ray emitting plasma. Current spectral models do not fully account for such nonequilibrium states and may lead to inaccurate measurements of the plasma properties such as temperature and ionization state, which in turn affect the mass ratios of iron-group elements.

In this dissertation, we aim to address the progenitor problem of SNe Ia by (1) constructing a self-consistent model of nonequilibrium plasma and (2) applying it to the first-ever high-resolution X-ray observations of a galactic Type Ia SNR, 3C 397, using the Resolve micro-calorimeter onboard the X-ray Imaging and Spectroscopy Mission (XRISM). We develop a spectral model that accounts for the nonequilibrium ionization and temperature of the plasma and compare it with the traditional models that assume a constant electron temperature. Our self-consistent model provides more accurate measurements of the plasma properties, such as temperature, ionization state, and elemental abundances. Then, we analyze the data of 3C 397 obtained by XRISM to investigate its mass ratios of iron-group elements. Our spatially-resolved analysis of 3C 397 reveals the detection of the Ti K-shell line in the southeast region of the remnant with more than $5\sigma$ significance that is higher than the previous studies. Such a detection of the Ti K-shell

line in the local region of the remnant suggests that the efficient electron captures occurred during the explosion of the innermost core of the progenitor WD with a high central density of $\gtrsim 5 \times 10^9$ g cm$^{-3}$. We also find the enhancement of the Ni/Fe mass ratio in not only the southeast region but also all regions of the remnant. The global enhancement of the Ni/Fe mass ratio in 3C 397 suggests that the neutronization degree of the progenitor WD was substantially high at the time of the explosion. Attributing this high neutronization to the metallicity of the progenitor WD would imply an unreasonably high progenitor metallicity, approximately three times the solar value. Alternatively, we consider electron capture processes leading up to the explosion, which could contribute to the increased neutronization degree. This process, known as the Urca process, also facilitates the high central density inferred from the detection of Ti K-shell emission lines. However, theoretical predictions indicate that the degree of neutronization achieved through the Urca process alone is insufficient to account for the observed values.

The exclusive detection of Ti in 3C 397 among the 17 Type Ia SNRs observable in X-rays suggests that progenitors like those of 3C 397 contribute only a small fraction to the overall population of SNe Ia. We estimated the fraction of the near-$M_{ch}$ mass WD progenitors with high central density to be up to 30% of the overall population of SNe Ia based on the $^{50}$Ti/$^{56}$Ni mass ratio. Additionally, we assess the detectability of neutrino emission from the electron captures during the explosion of the near-$M_{ch}$ mass WD progenitors. We find that the neutrino emission could be detected near the Milky Way by the current and future neutrino detectors, such as the Super-Kamiokande and Hyper-Kamiokande experiments and the neutrino flux could be a useful probe to constrain the central density of the progenitor WDs.