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Entrance examination – Question sheet (Physics)
Department of Space and Astronautical Science,
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Question 1

(1) Assume that there is a spherical body with the radius of R, the mass of M, and uniform density. Express the gravitational acceleration g that acts in the following cases on a point mass with the mass of m which is located in the distance of r from the center of the spherical body by using g_0, R, and r, where g_0 is the gravitational acceleration on the surface of the spherical body.

（a）When the point mass is located outside of the spherical body ($r > R$)
（b）When the point mass is located inside of the spherical body ($r < R$)

In the calculation, use the following formulae of the gravitational force $f(r)$ which a spherical shell with the radius of r_c, mass of dM_c, and minute thickness applies on the point mass with the mass of m that is located in the distance of r from the spherical shell's center O as shown in Figure 1-1:

$$\begin{cases} \text{When } r > r_c: & f(r) = \dfrac{Gm\ dM_c}{r^2} \\ \text{When } r < r_c: & f(r) = 0 \end{cases}$$

(where G is the universal gravitational constant)

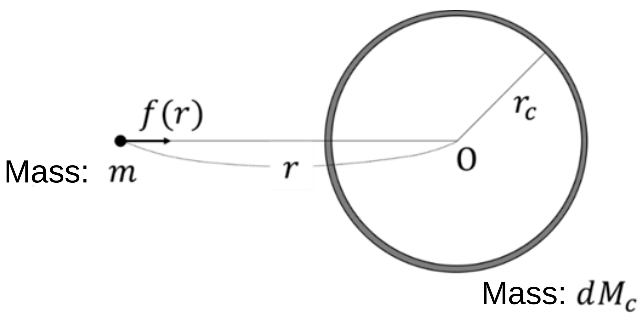

Figure 1-1: The spherical shell with minute thickness (sectional diagram) and the point mass.

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(2) Assume to apply an initial velocity to the point mass in the previous item (1) that is located close to the surface of the spherical body to allow it moving along the great circle without falling onto the spherical body. Express the period of the movement by using g_0 and R.
Assume $m \ll M$ and it is allowed to ignore the other force than the gravitational force applied by the spherical body.

(3) A straight through hole with a small diameter is bored through the center of the spherical body in the item (1) (Figure 1-2). Describe the movement of the point mass when it is placed immediately above the entrance of the through hole and then released quietly. Describe the characteristics of the movement in comparison with that in the case in the item (2).
Assume $m \ll M$ and it is allowed to ignore the other force than the gravitational force applied by the spherical body.

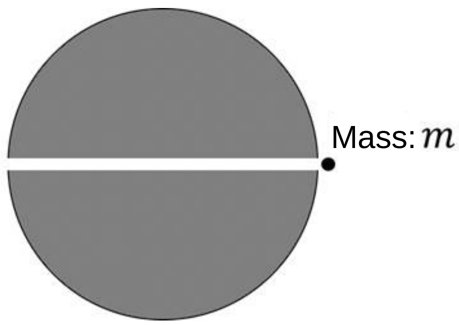

Figure 1-2: The spherical body with a through hole (sectional diagram) and the point mass placed above the entrance of the through hole.

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Question 2

For a plane wave that propagates through a wide free space, the pointing vector S, electric field E, and magnetic field H constitute the relation described by the following formula:

$S = \boxed{}$ (2-1)

E, H, and S constitute a right-handed system in the order of listing.

(1) Write down the formula (2-1) in the appropriate answer column.

The ratio k of the size of the electric field E and magnetic field H is expressed as follows by using the dielectric constant ε and magnetic permeability μ of the space:

$k = \boxed{}$ (2-2)

(2) Write down the formula (2-2) in the appropriate answer column.

In addition to that, the angular frequency ω, wave length λ, and the speed of light c have the following relation:

$\frac{\omega\lambda}{c} = \boxed{}$ (2-3)

(3) Write down the formula (2-3) in the appropriate answer column.

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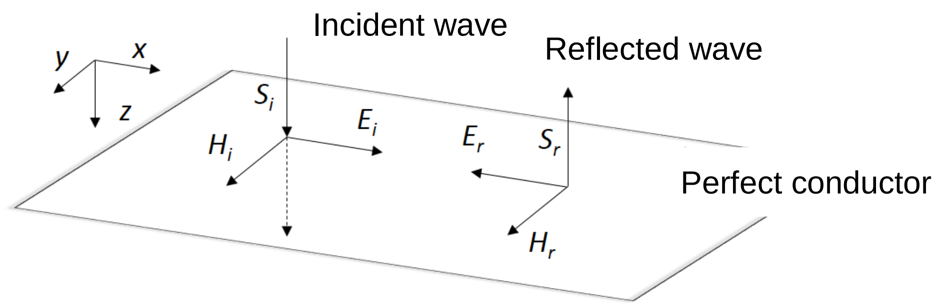
Consider the standing wave generated in the case when a plane sinusoidal electromagnetic wave vertically enters a perfect conductor (of which the electric conductivity is infinite). The figure below shows the electric field E_i, magnetic field H_i, and pointing vector S_i of the incident wave, and the electric field E_r, magnetic field H_r, and pointing vector S_r of the reflected wave. The surface of the perfect conductor is located at $z = 0$. The electric field and the magnetic field are respectively located in the x and y direction.



The electric field E_i of the incident wave at the position z is expressed in the following formula by using the electric field strength A_i, angular frequency ω, the speed of light c, and time t:

$$E_i = A_i \sin \omega \left(t - \frac{z}{c}\right) \qquad \text{(2-4)}$$

Similarly, the electric field of the reflected wave E_r at the position z is expressed in the following formula by using the electric field strength of the reflected wave A_r:

$$E_r = A_r \sin \omega \left(t + \frac{z}{c}\right) \qquad \text{(2-5)}$$

The electric field outside the conductor is expressed in the following formula:

$$E_i + E_r = A_i \sin \omega \left(t - \frac{z}{c}\right) + A_r \sin \omega \left(t + \frac{z}{c}\right) \qquad \text{(2-6)}$$

Because no electric field is generated on the surface of a perfect conductor, the boundary condition for the surface of the conductor $(z = 0)$ is expressed in the following formula:

$$E_i + E_r = \boxed{} \qquad \text{(2-7)}$$

(4) Write down the formula of the boundary condition (2-7) in the appropriate answer column.

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Based on the condition determined in the formula (2-7), the electric field outside the conductor can be expressed by using the electric field strength of the incident wave A_i:

$$E_i + E_r = \boxed{\text{(i)}} \qquad \text{(2-8)}$$

(5) Select the formula that represents the part of $\boxed{\text{(i)}}$ of the formula (2-8) from the followings and write it down in the appropriate answer column.

$$A_i \sin\frac{\omega z}{c}\cos\omega t, \quad -2A_i \sin\frac{\omega z}{c}\cos\omega t, \quad -\frac{A_i}{2}\sin\frac{\omega z}{c}\cos\omega t, \quad -A_i \sin\frac{\omega z}{c}\sin\omega t$$

(6) Illustrate the standing wave of the electric field in the appropriate answer column. Indicate the standing wave within 1.5 wavelength distance from the surface of the conductor while clearly describing the location of the loop and node and the period. Also describe the reason for the position of the node in the relation to the surface of the conductor.

On the other hand, the magnetic field of the incident wave is expressed in the following formula by using the ratio k of size of the electric field E and magnetic field H:

$$H_i = kE_i = kA_i \sin\omega\left(t - \frac{z}{c}\right) \qquad \text{(2-9)}$$

Similarly, the magnetic field of the reflected wave is expressed in the following formula while considering that the pointing vector is directed to the opposite direction:

$$H_r = -kE_r = \boxed{\text{(ii)}} \qquad \text{(2-10)}$$

(7) Select the formula that represents the part of $\boxed{\text{(ii)}}$ of the formula (2-10) from the followings and write it down in the appropriate answer column.

$$kA_i \sin\omega\left(t - \frac{z}{c}\right), \quad -kA_i \sin\omega\left(t - \frac{z}{c}\right), \quad kA_i \sin\omega\left(t + \frac{z}{c}\right), \quad -kA_i \sin\omega\left(t + \frac{z}{c}\right)$$

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Therefore, the magnetic field outside the conductor is expressed in the following formula by using the electric field strength of the incident wave A_i:

$$H_i + H_r = \boxed{\text{(iii)}} \qquad (2\text{-}11)$$

(8) Select the formula that represents the part of $\boxed{\text{(iii)}}$ of the formula (2-11) from the followings and write it down in the appropriate answer column.

$$kA_i \sin\frac{\omega z}{c}\cos\omega t,\quad 2kA_i \sin\frac{\omega z}{c}\cos\omega t,\quad 2kA_i \cos\frac{\omega z}{c}\sin\omega t,\quad \frac{kA_i}{2}\sin\frac{\omega z}{c}\cos\omega t$$

(9) Illustrate the standing wave of the magnetic field in the appropriate answer column. Indicate the standing wave within 1.5 wavelength distance from the surface of the conductor while clearly describing the location of the loop and node and the period. Also describe the reason for the position of the node in the relation to the surface of the conductor.

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