Applicant number: Name:

Entrance examination – Question sheet (Mathematics)
Department of Space and Astronautical Science,
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Question 1-1

Find the general solution of the following differential equation, where c is an arbitrary constant.

Where: $y' = \frac{dy}{dx}$.

(1) $y' = \dfrac{y-1}{xy}$

(2) $x^2 y' - (y^2 + xy) = 0$

(3) $y' - \dfrac{y}{x} = x^2$

Question 1-2

For $u(x,t)$, consider the following partial differential equation, where c is a positive constant.
Answer the following questions:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

(1) Derive the following formula by using the following two independent functions: $\xi = x + ct$, $\eta = x - ct$.

$$\frac{\partial^2 u}{\partial \xi \partial \eta} = 0$$

(2) Show that the general solution of the formula derived in the previous item (1) is indicated as follows, where ϕ and ψ are arbitrary functions.

$$\mathrm{u}(x,t) = \phi(x+ct) + \psi(x-ct)$$

(* This material is collected after the exam.)

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(3) For the general solution indicated in the previous item (2), let the initial conditions be as follows:

$$u(x,0) = f(x), \qquad \left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} = F(x)$$

Show that the solution is expressed as follows:

$$u(x,t) = \frac{1}{2}\big(f(x+ct) + f(x-ct)\big) + \frac{1}{2c}\int_{x-ct}^{x+ct} F(y)dy$$

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Question 2

(1) Answer the following questions regarding linear independence / dependence of a vector.

(i) Find the value of the real number s when the following two vectors are linearly dependent.

$$\boldsymbol{a}_1 = \begin{pmatrix} 4 \\ 10 - 2s \end{pmatrix},\ \boldsymbol{a}_2 = \begin{pmatrix} 5 + s \\ 12 \end{pmatrix}$$

(ii) Answer whether the following three vectors are respectively linearly independent or dependent. Also state the reason for your answer. In addition to that, when the three vectors respectively represent a position of a point in the three-dimensional space, briefly indicate the positional relation among those three points.

$$\boldsymbol{b}_1 = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix},\ \boldsymbol{b}_2 = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix},\ \boldsymbol{b}_3 = \begin{pmatrix} -15 \\ 15 \\ -14 \end{pmatrix}$$

(2) Find the eigenvalue and the characteristic vector of the following matrices.

(i) $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

(ii) $B = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$

(3) A secondary square matrix can be diagonalized when it has two characteristic vectors that are linearly independent. Assume that the matrix B in the item (2)(ii) is diagonalized as follow:

$$C = P^{-1}BP$$

Find the matrix P and the diagonal matrix C.

(4) The matrix exponential of the matrix D is defined as follows by using the expansion of the exponential function:

$$e^D = \sum_{n=0}^{\infty} \frac{D^n}{n!}$$

Calculate e^B for the matrix B in the item (2)(ii) by using the result of the item (3).

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