# 3-Dimensional ionospheric GPS tomography over Japan, simulation and data analysis results

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## 1. ***Abstract***:

A new 3-dimensional GPS ionospheric tomography technique is being developed by Kyoto University which will not require an ionospheric model as initial guess. This method is different to most ionospheric tomographic techniques as they require a background ionospheric model as an initial guess that could bias the resulting electron density tomogram. This technique rather uses a prior condition that the electron density should not exceed a certain value that is determined by the restrained parameter. The tomography technique uses constrained Least-squares fit to derive electron density distribution even in disturbed conditions.

This paper presents some preliminary results that are obtained by using this new tomographic technique. In the present study, the three dimensional tomogram images are produced using the GPS data and are compared with simulated tomograms obtained using the synthetic GPS-TEC data produced using NeQuick model [Radicella and Leitinger, 2001].

## 2. ***Introduction***:

Ionospheric total electron content (TEC) measurements have re-gained importance in the recent years since there was an exponential usage and growth of navigation applications using Global Positioning System (GPS) in various fields. Delay from the ionospheric electron content and irregularities are among the major factors affecting the trans-ionospheric navigation and communication systems. Ionospheric Tomography technique is used to estimate the electron density distribution in the ionosphere. Hence the study of three dimensional distribution of ionospheric electron density is essential to study the ionospheric phenomena that effect the radio wave propagation. Also, imaging the vertical structure of the ionosphere could lead to better scientific understanding of this region.

Austen et al, 1988 suggested two dimensional tomographic technique using the TEC data from beacons from LEO, which applies medical tomography to study the ionosphere. Ionospheric observations are limited by the minimum elevation angle, rate of data collected, and the number and spacing of receivers in the array. The tomography technique requires the knowledge of number density, and is also insensitive to stratified ionosphere. Hence, a prior data has to be included from other sources; the reconstructed number density is only accurate up to the class of background ionosphere. Austen et al. [1986; 1988] used the line-of-sight TEC data from naval navigational satellite system (NNSS), flying around 1,100 km altitude, to reconstruct the two-dimensional structure of the ionospheric electron density. However, this method can just reconstruct the two-dimensional electron density along the satellite-to-receiver flying plane.

The GPS has more than 24 satellites around 20,200 km altitude in 6 orbital planes whose inclination angle is 55 degree, which can provide the global coverage of observations. The TEC value along the ray path between the GPS satellite and GPS receivers can be measured by the dual-frequency ($f_1$ = 1.57542 GHz and $f_2$ = 1.22760 GHz) GPS signals continuously. Good horizontal spatial and temporal resolution of TEC distribution can be obtained by taking the advantage of the dense GPS network of Japan's GPS Earth Observation Network (GEONET), which is operated by Geographical Survey Institute (GSI). And the vertical structure of electron density is also important to study the ionospheric features. Therefore, the tomography technique is used to reconstruct the three-dimensional electron density structure using GPS-TEC data. However, because of the geometric limitation of GPS observation, it is hardly to get the vertical structure of electron density by previous methods of tomography. In this paper, a new GPS tomography technique that was developed and the preliminary results will be presented in the following section.

## 3. *Tomography Technique*:

In this study, a new GPS tomography method to reconstruct the ionospheric three-dimensional electron density structure was developed with the restrained least-square method. This ionospheric tomography technique is different to most techniques as they require a background ionospheric model as an initial guess that could bias the resulting electron density image. This technique rather uses a prior condition that the electron density should not exceed a certain value that is determined by the restrained parameter. The tomography technique uses constrained Least-squares fit to derive electron density distribution even in disturbed conditions. The three-dimensional space with the altitude form the 80 km to 20,200 km, around the height of GPS satellite, was divided in small grids as shown in Figure 1. The GPS-TEC observation data along the ray path from a GPS satellite to a GPS receiver can be written as:

$$\sum_j A_{ij} x_j = b_j \qquad \text{---- (1)}$$

Where $A_{ij}$ is a m × n matrix which indicates the length of path $i$ in grid $j$, $i$ = 1,…,m and $j$ = 1,…,n. $x_j$ indicates the electron density in grid $j$. $b_i$ is the TEC value along one GPS observational path i. In order to solve the equation (1), the least-square fitting with a restrained condition was used. The cost function, $J(x)$, with restrained least-square fitting was defined as,

$$J(x) = \|b - Ax\|^2 + \lambda \|Wx\|^2 \qquad \text{----- (2)}$$

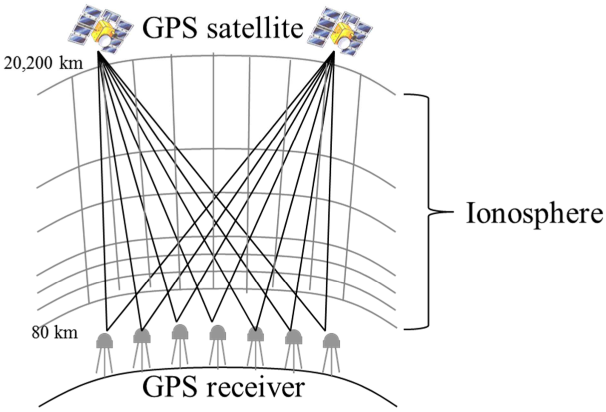

Fig. 1 *Schematic figure of observation path between GPS receiver and GPS.*

The first term in the equation (2) is the normal least square fit and the second term is the restrained condition. Where $W$ is the weight matrix containing the restrained parameter whose weighting changes with altitude. This restrain parameter is dynamically derived from NeQuick model and dependent on the date and time of GPS data being used. Since the electron density varies a lot around the F-region, a weak restrain parameter is applied from 150 km to 700 km to allow room for the electron density to change independent of the model derived restrain parameter. And the value λ, called as hyper parameter, indicates the weighting of the restrained condition comparing with the least-square term. The change in hyper parameter changes the importance is given for the restrained condition term in the cost function. The algorithm finds the optimum value of hyper parameter to obtain solution.

The data used for the tomography include the dense GPS network of Japan's GEONET and the synthetic GPS data derived from NeQuick model. The region chosen for GPS tomography include range of 120°E to 150°E longitudes, 20°N to 50°N latitudes, with the grid resolution of 2° in latitudinal and longitudinal direction. The altitude resolution is 20 km from 80 to 600 km, from there 50 km resolution up to 1000 km and then 500 km resolution up to to 2000 km height. And about 748 GPS receiver locations were chosen in each 0.25° × 0.25° grid over the Japanese region for the current GPS tomography study.

## 4. *Results*:

The Figure 2 shows the comparison between the electron density profiles from the NeQuick model and the simulated tomogram result for 23rd May, 2012 at 03:30 UT. It can be seen that result density profiles closely follows the model profile. This simulation shows the ability of the method to reproduce the profile from the synthesized GPS-TEC data derived from the model.

The vertical profile (two-dimensional latitude-altitude structure) of electron density can be seen from Figure 3 (a) model, (b) simulated result, (c) the associated error distribution, and (d) the path count in each grid box along the 136°E longitude plane.

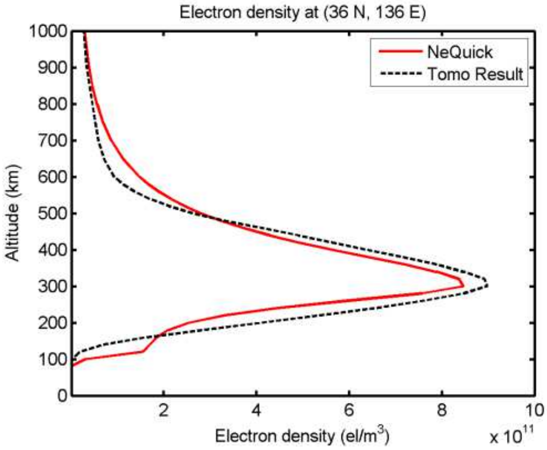

Fig. 2 *Electron density profile from (a) NeQuick model and (b) tomogram result.*

From figures 3(a) and (b), it can be seen that the simulated tomography results showed a similar distribution of the electron density with that of the input NeQuick electron density. The reconstructed electron density error shown in Figure 3(c) indicates a well reproduction of the model, except the areas where number of observational paths are low as seen from figure 3(d).

And the figure 4(a) and (b) shows the horizontal structure of the electron density at 300 km altitude on the latitude-longitude plane from the model and the simulated tomography result. It can be seen that they both had almost similar distribution, as also inferred be seen from the error difference between them from figure 4(c). From figure 4(d), the reconstruction is seen relatively good in all regions even where the path count falls moderately. Because of the geometric limitation of the GPS observation paths and lack of the horizontal observations, the simulated tomography image could not reconstruct the model electron density without error.

The actual GPS TEC values when given as input instead of simulated values also produced realistically good tomography images as shown in figures 5 and 6. The figures 5 and 6 show the latitude-altitude structure of electron density and horizontal distribution of electron density respectively. And the figures 5(c) and 6(c) show more difference between the model to the reconstructed tomography image than the simulated ones (figures 3c & 4c). Also some difference might arise because of the ambiguity that model itself will be different from the experimental or observed TEC values.

## 5. *Discussion*:

The preliminary results show that the 3 Dimensional GPS ionospheric tomography algorithm is reasonably good in the re-construction of the vertical and horizontal electron density images. The simulation results showed that the electron density was well re-constructed by the tomography algorithm, 80% of the reconstructed error was within the range of ±0.5 TECU. While with the observation data the re-constructed images has less accuracy compared with simulation, and about 50 – 70% of the reconstructed error was within ±0.5 TECU when compared with the model data. To verify the integrity of the method with observation data, these reconstructed values should be compared with the observed density values too, to avoid model to observational error. The future works include successful combination of GPS and Beacon data as inputs for tomography which will result in better tomography, increasing the resolution of the tomography imaging and also improve the algorithm to enable to produce near real time 3 dimensional ionospheric tomograms.

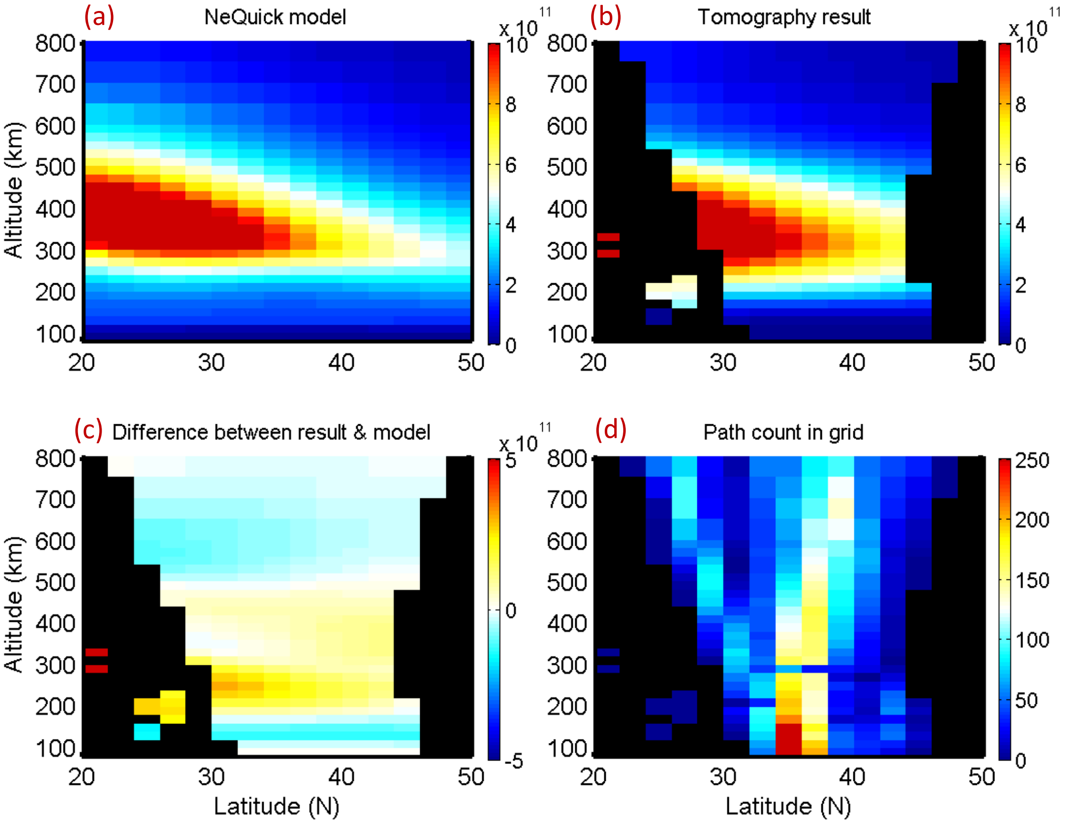

Fig. 3 *Two dimensional electron density structure of (a) NeQuick model (b) Simulated result (c) difference between them and (d) Path count in each grid, at 136° longitude.*

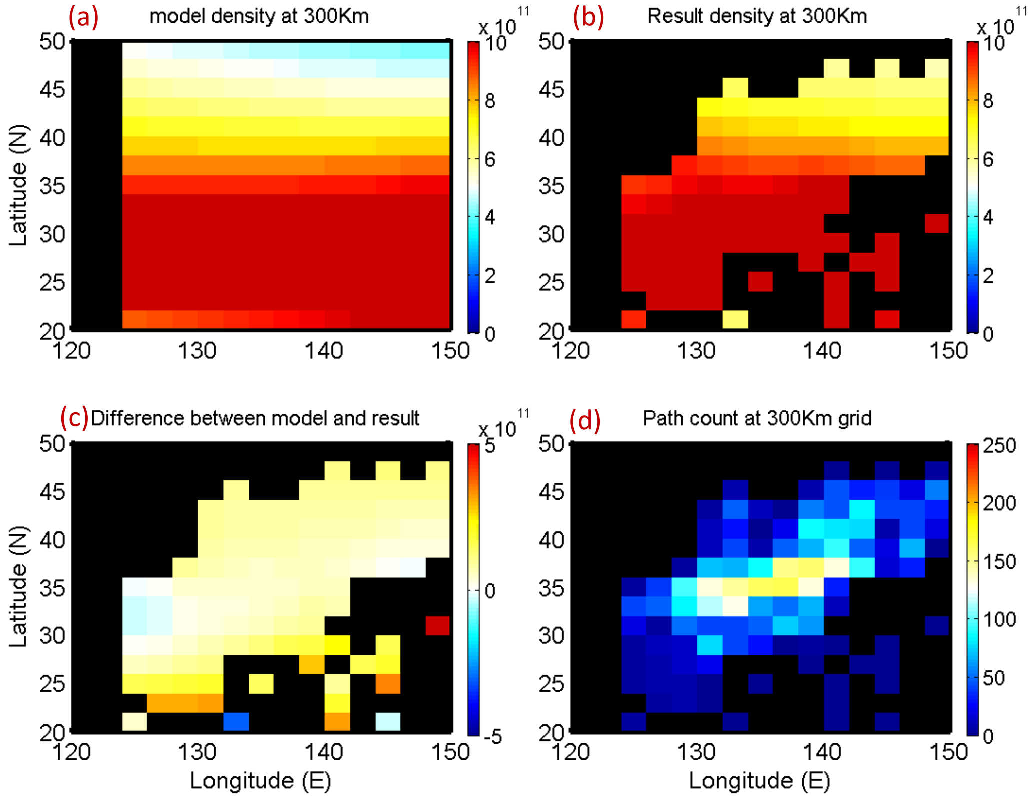

Fig. 4 *Horizontal structures of electron density of (a) NeQuick model, (b) simulated result (c) difference between them and (d) path count in each grid at 300 km altitude in latitude and longitude.*

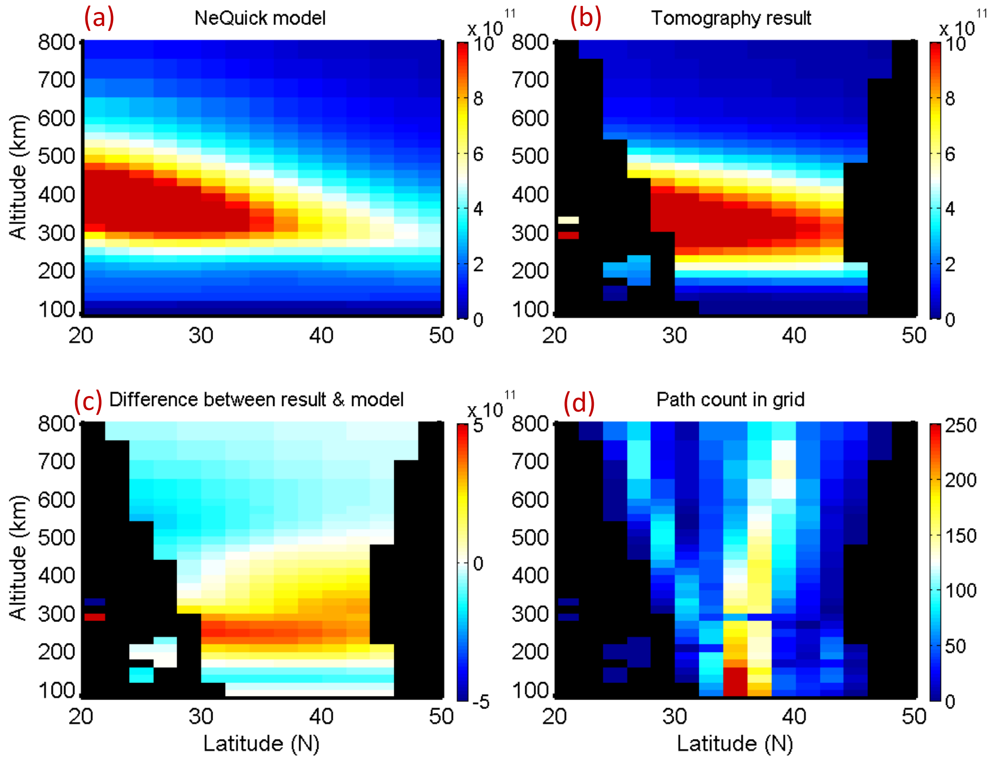

Fig. 5 *Two dimensional electron density structure of (a) NeQuick model (b) GPS-TEC result (c) difference between them and (d) Path count in each grid, at 136° longitude.*

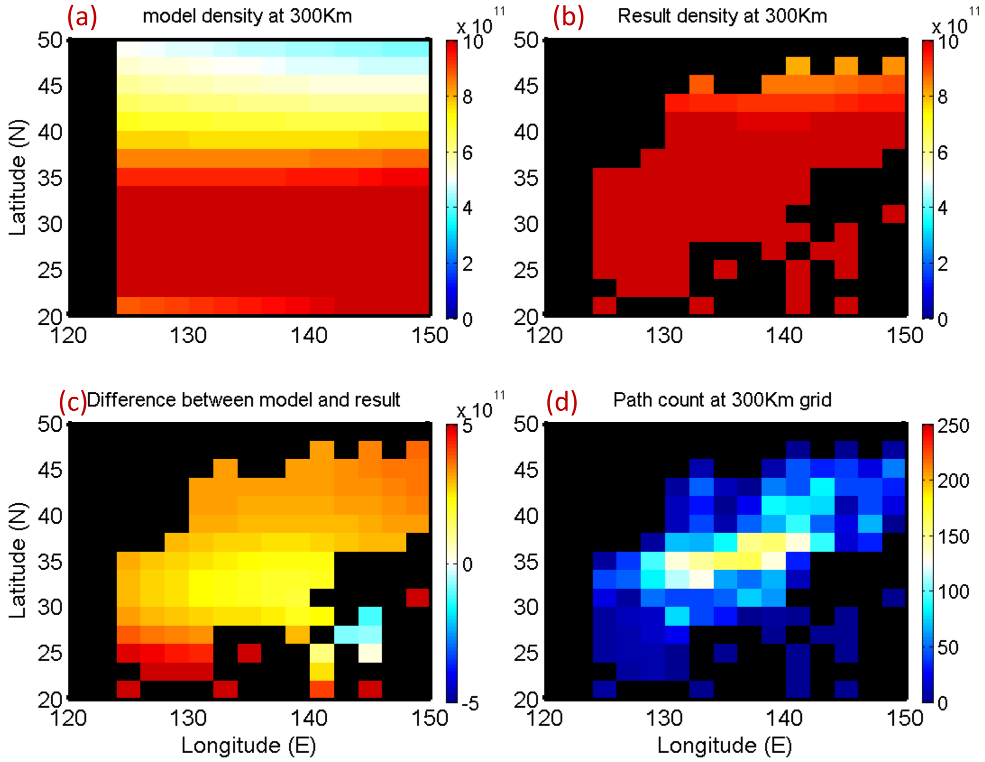

Fig. 6 *Horizontal structures of electron density of (a) NeQuick model, (b) GPS-TEC result (c) difference between them and (d) path count in each grid at 300 km altitude in latitude and longitude.*

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