A brief introduction to helio- and asteroseismology

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Helio- and asteroseismology

- A quick overview
  - What they are
  - Why they are interesting
  - What we have learned
- How do they work?
  - More physical and mathematical details
- Some more recent results

A quick overview

Helio- and asteroseismology

- Investigation of the solar and stellar interiors based on their oscillations
  - The Sun and stars are not transparent to electromagnetic waves
  - They are transparent to ‘waves’
  - Seismic approaches are the only way to study inner work of the Sun and stars

Helio- and asteroseismology

- Oscillations?
  - Variable stars are known from ancient times and many of them are pulsating variables
  - Since 1962, the Sun is also known to be a pulsating variable
    - Leighton et al (1962)'s discovery of the 5-minute oscillations

Global vs. Local

- Two main flavours of helioseismology
  - Global helioseismology
    - based on global eigenfrequencies
    - for ‘global’ or highly symmetric structures
  - Local helioseismology
    - based on (for example) local travel times
    - for localized measurements
- Asteroseismology can only be ‘global’

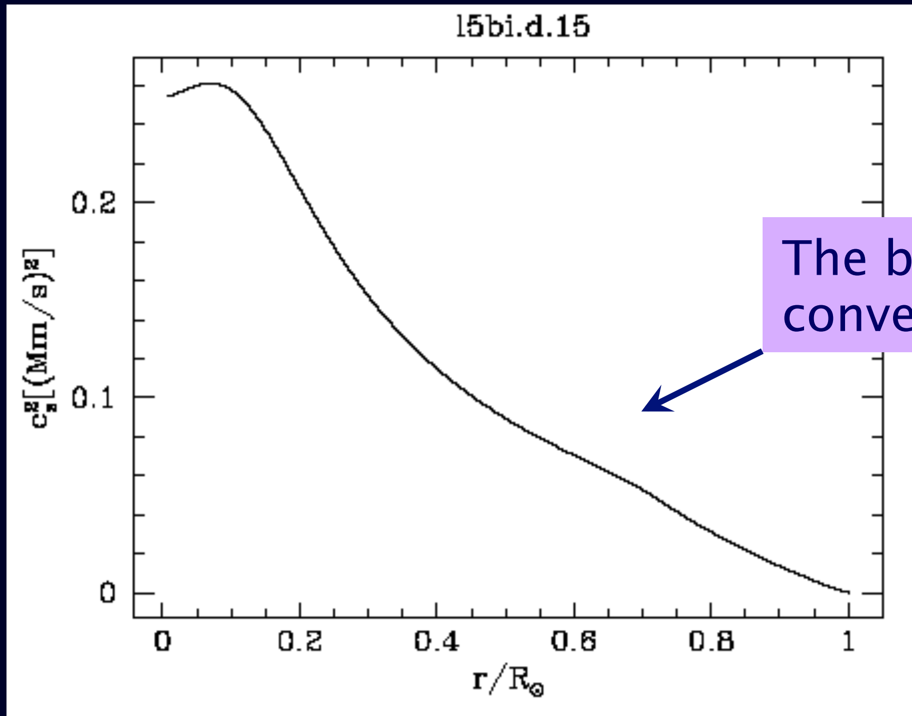
What do we want to study?

- Is the standard solar model correct?
  - Do we understand solar/stellar evolution processes?
- Ingredients of solar dynamo
  - Differential rotation, meridional flow
  - Convection
- How the internal processes are connected to observable surface phenomena
  - Subsurface ‘weather’
  - Flux emergence processes

Soundspeed inversion

- The depth of the convection zone is about 20 Mm, not 15 Mm
  - The first major result from helioseismology

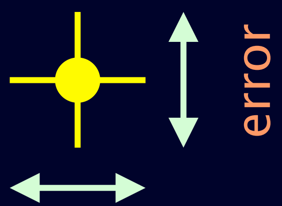
$$c^2 \propto \frac{T}{\mu}$$

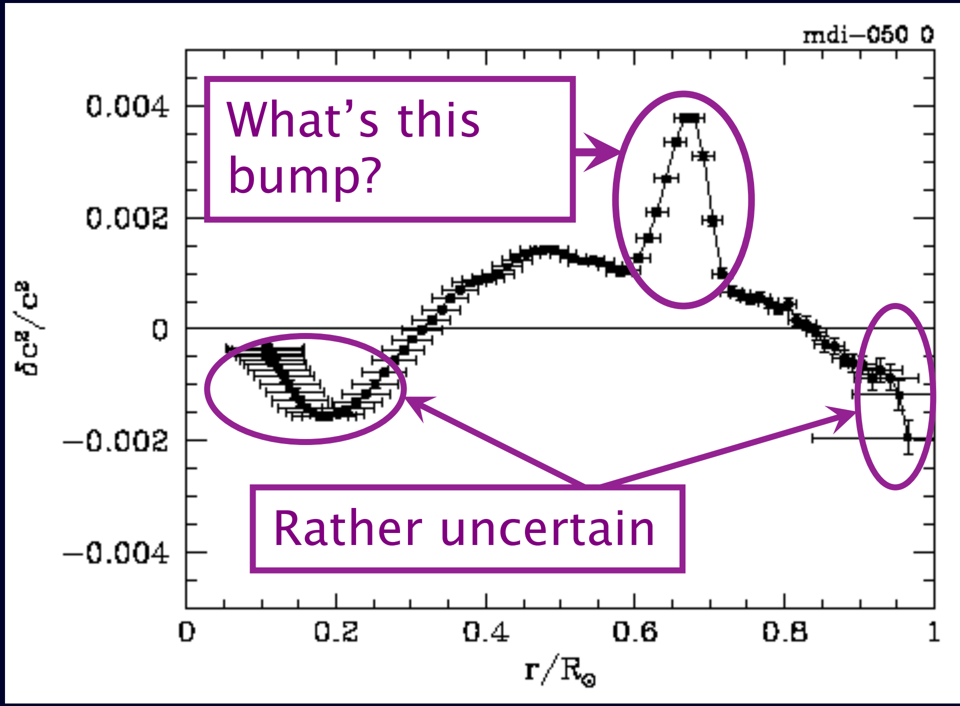


The base of the convection zone

Soundspeed inversion

- This modern model agrees with the 'observation' within a half per cent accuracy





Solar differential rotation

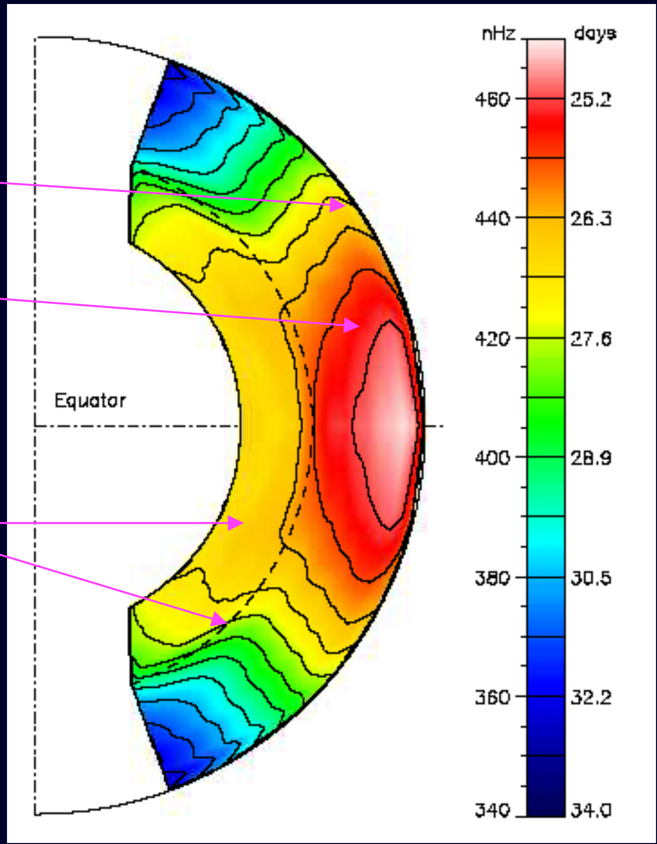
- Based of measurement of rotational shifts of eigenfrequencies

Surface shear layer

Non-columnar rotation

'Tachocline'

Rigid rotation?

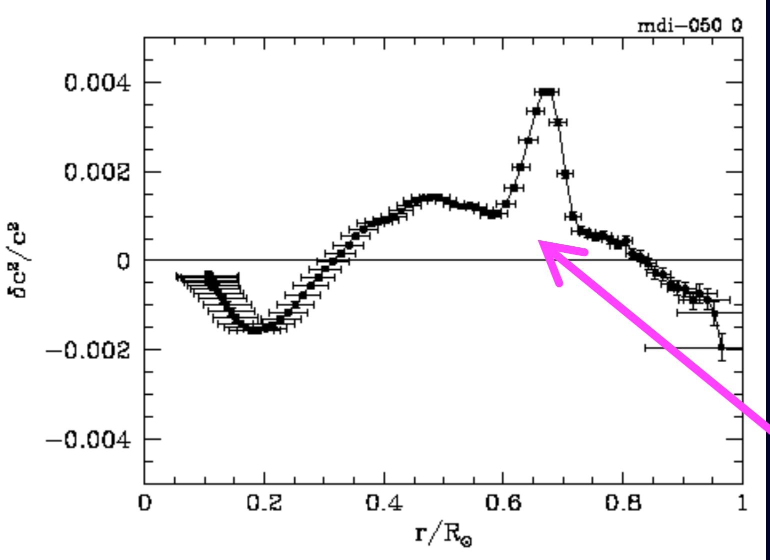


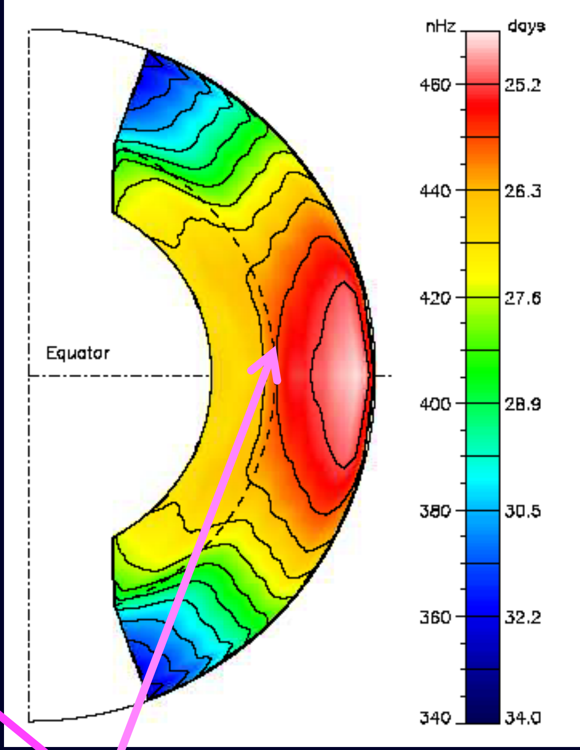
Previous dynamo calculations all rejected

Tachocline

- A steep gradient in the rotation rate

Dynamo action?



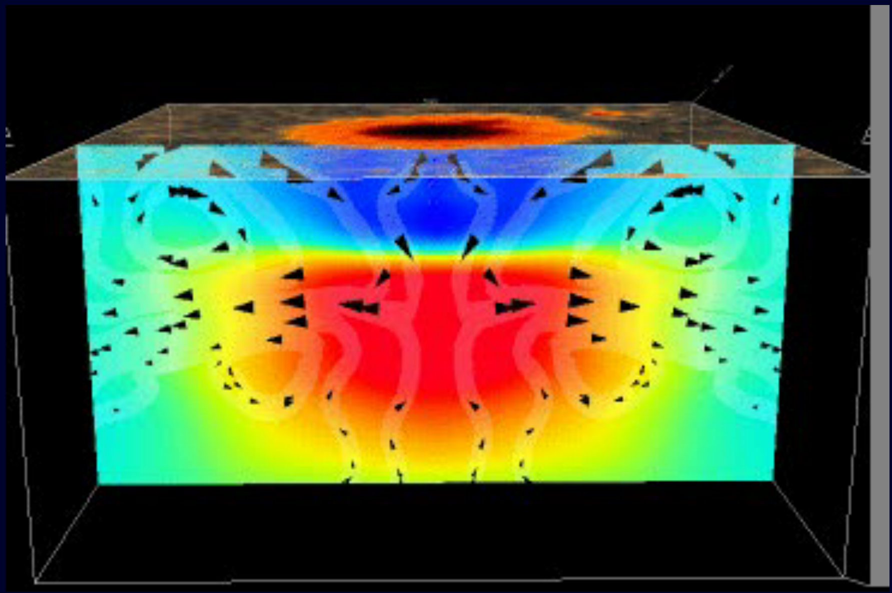


Shear-induced mixing counteracting gravitational settling of Helium?

Tachocline

Structure around a sunspot

- From time-distance method



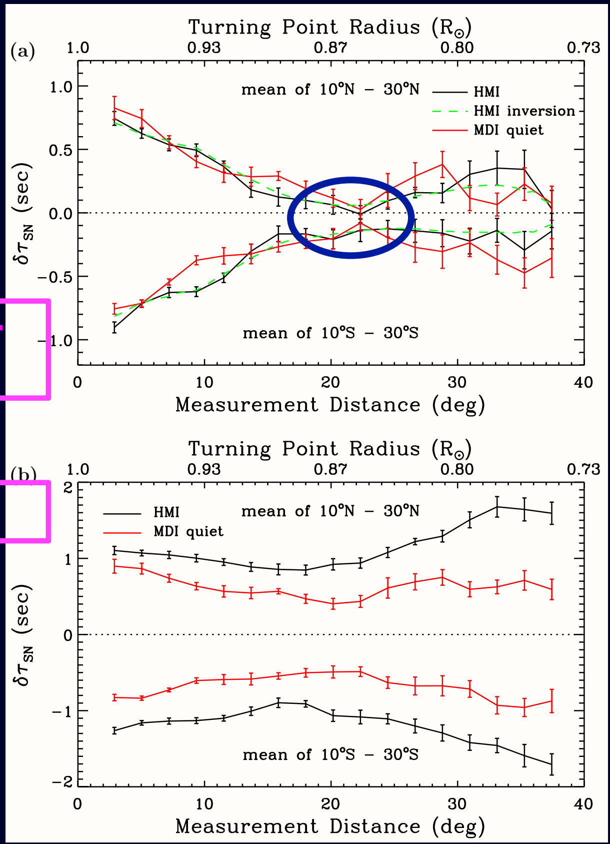
A very well-known result but
nobody is sure if this is correct

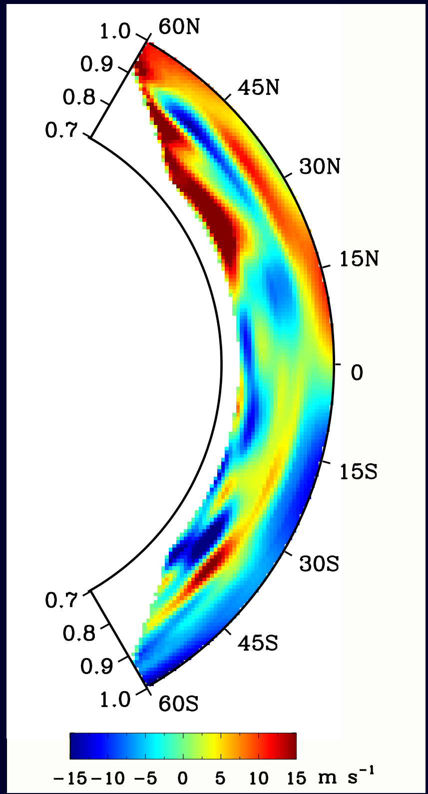
Double-cell meridional flow?

- Zhao et al. (2013)

Corrected for CLV

Uncorrected



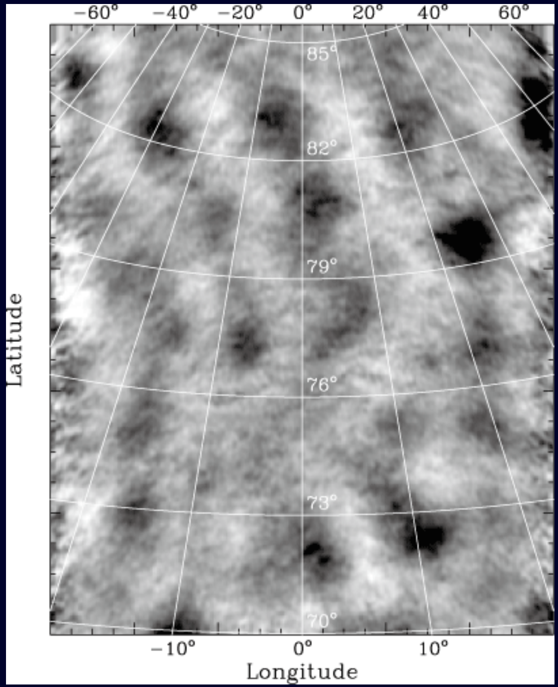


Supergranulation

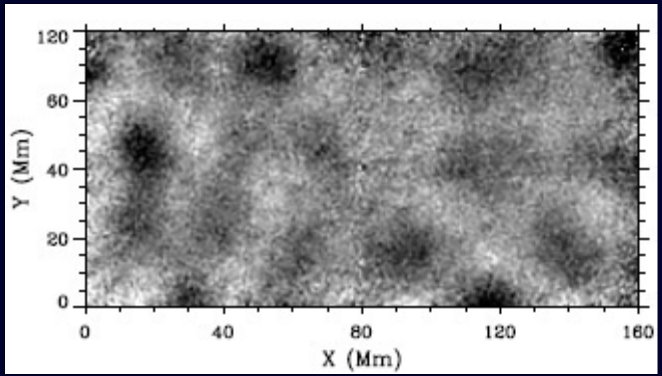
□ 'Divergent' flow signatures

Travel-time difference (inward/outward) maps (15-Mm scale)

Polar region
(Nagashima et al. 2011)



Equatorial
(Sekii et al. 2007)

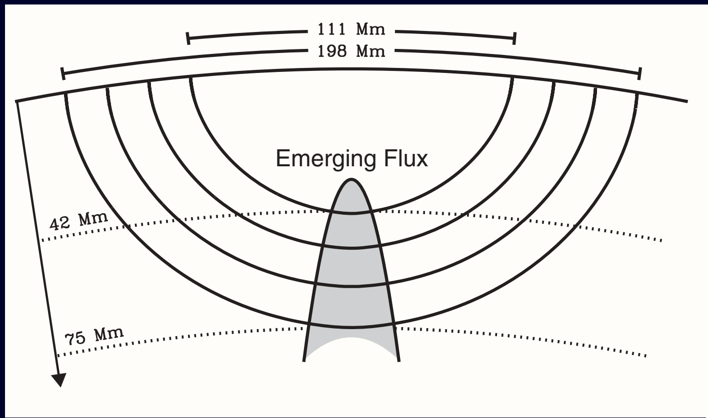


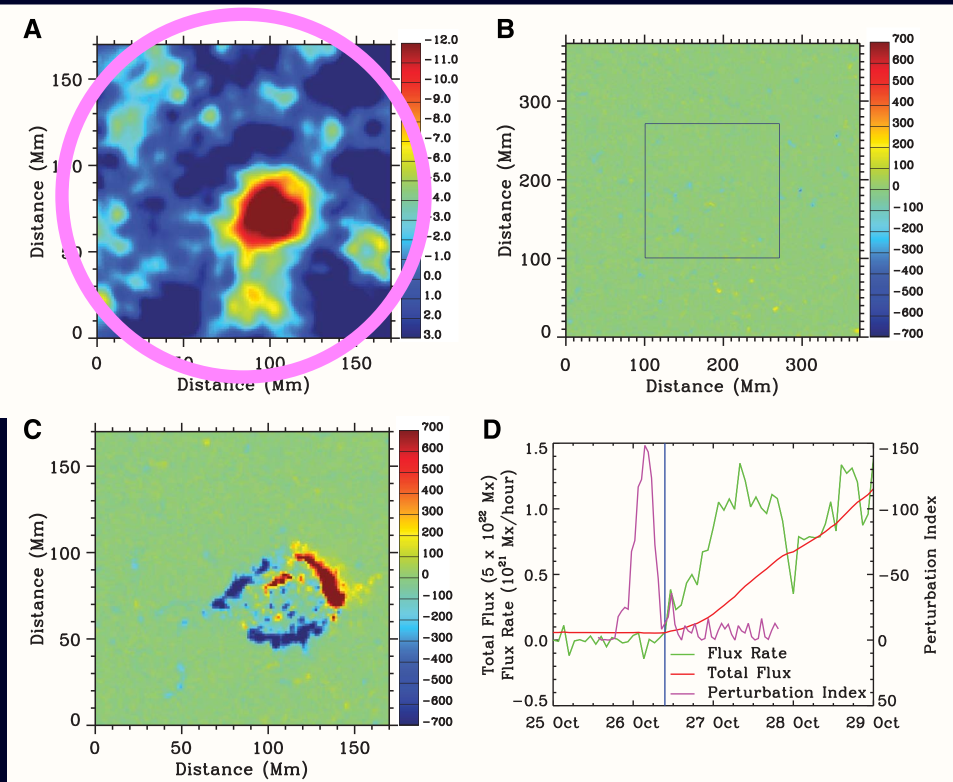
Supergranulation

- Helioseismic analyses are finding out that supergranulation is a fairly ‘shallow’ phenomenon
  - Duvall 1998 (MDI), Sekii et al 2007 (Hinode/SOT)
  - ~10Mm deep, give or take
- Polar supergranules are smaller and deeper than their low-latitude counterparts? (Nagashima 2010)

Emerging flux detections

- Can we detect magnetic fluxes before their emergence?



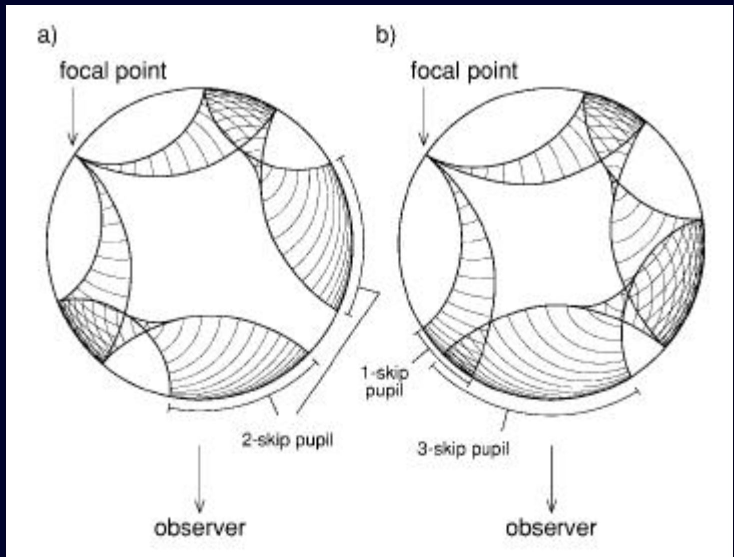


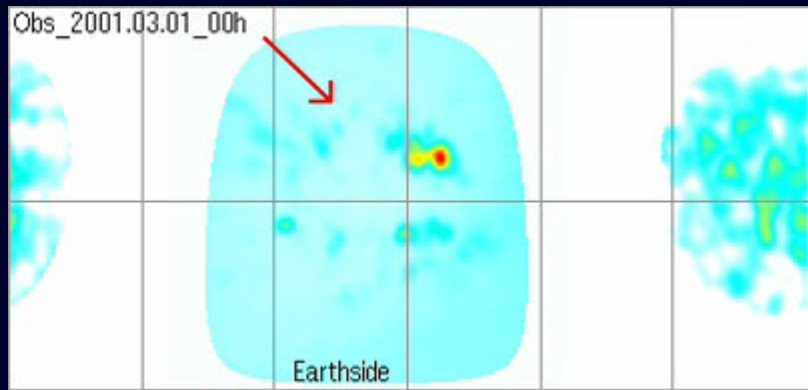
A travel-time perturbation map

Ilonidis, Zhao & Kosovichev (2011)

Far-side imaging

- The other side of the sun

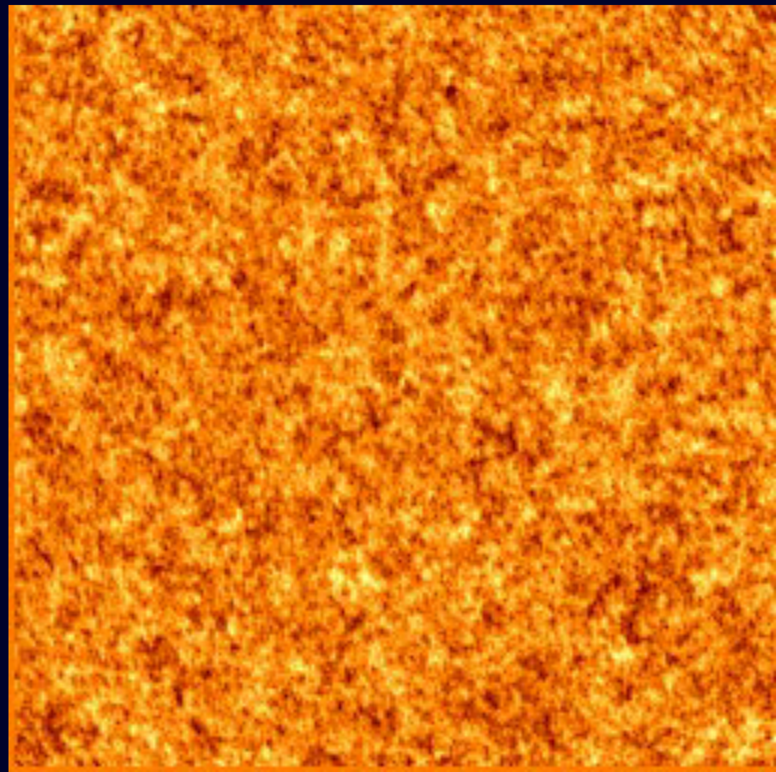




How do they work?

Dopplergram movies

- From SOHO/MDI

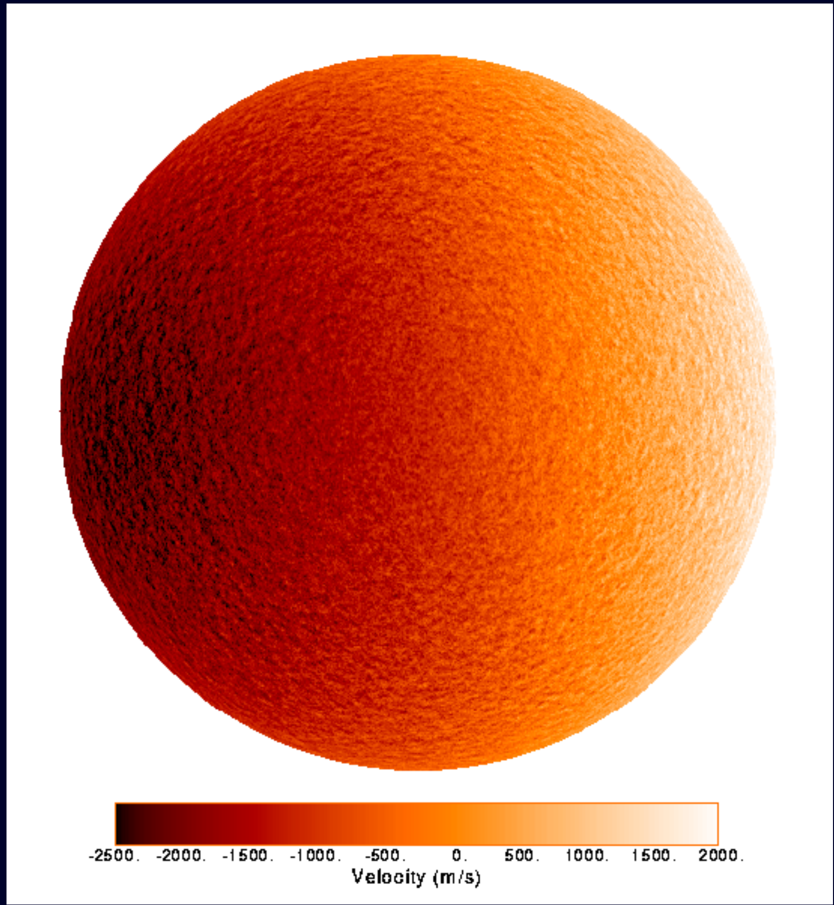


Dopplergram

- Dopplergram obtained by SOHO/MDI

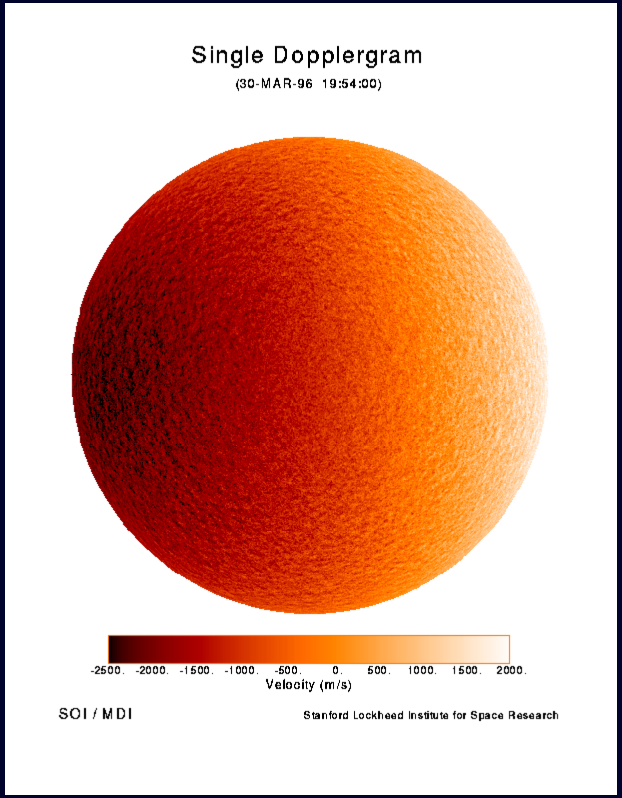
$-2\mathrm{km/s} < v < 2\mathrm{km/s}$

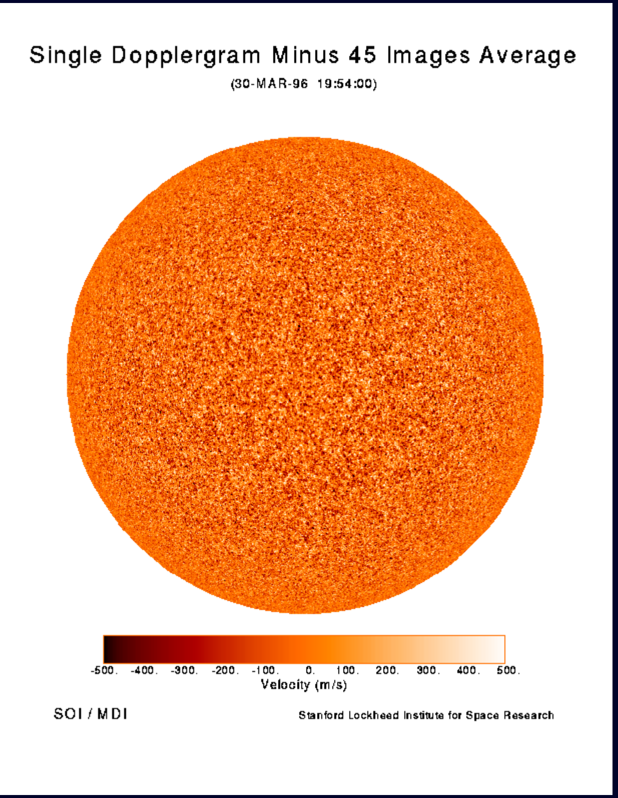
Dominated by solar differential rotation



Dopplergram

- By subtracting 45-min average we can filter out rotation and supergranulation





FSH decomposition

- Any scalar function on sphere can be expanded in spherical harmonics

$$f(\theta,\phi) = \sum_{lm} f_{lm} Y_l^m(\theta,\phi)$$

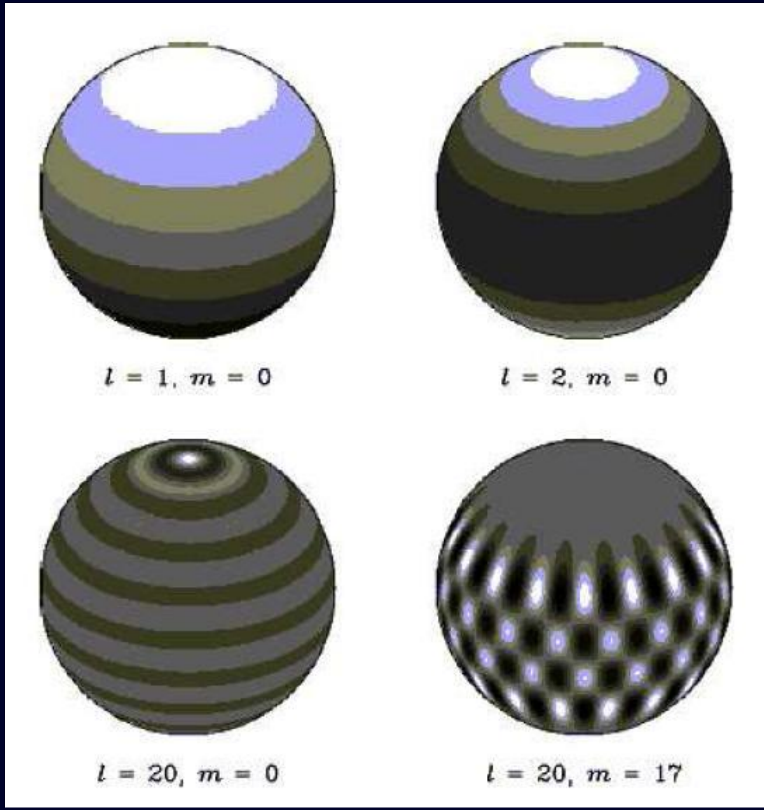
$$Y_l^m(\theta,\phi) = P_l^m(\cos\theta) e^{im\phi}$$

l : degree

m : azimuthal order

$f(\theta,\phi)$: symmetric

$\Rightarrow f_{lm}$ indep't of m



FSH decomposition

- For simplicity, we assume we observe the radial velocity (rather than the line-of-sight velocity)
- In spatial domain, the velocity field can be expanded in spherical harmonics

$$v(\theta,\phi,t) = \sum_{lm} A_{lm}(t) Y_l^m(\theta,\phi)$$

$v(\theta,\phi,t)$: radial velocity field

$Y_l^m(\theta,\phi)$: spherical harmonic function with degree l and azimuthal order m

FSH decomposition

- In time domain, Fourier decomposition comes in handy

$$A_{lm}(t) = \int a_{lm}(\omega) e^{i\omega t} d\omega$$

- Then we have Fourier-Spherical-Harmonic decomposition of the velocity field

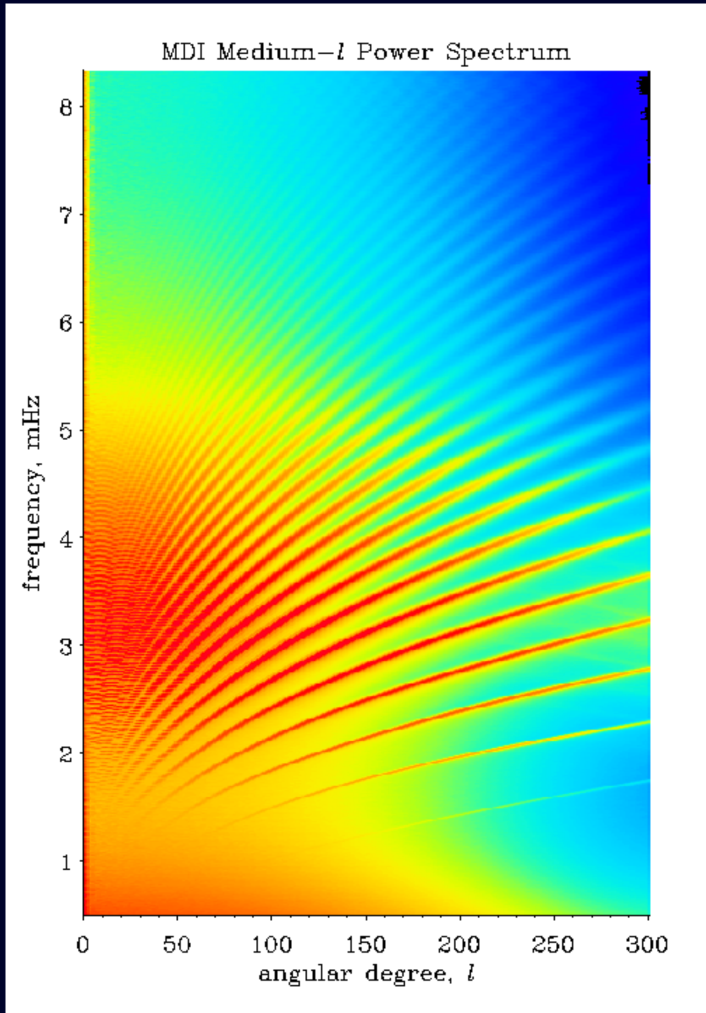
$$v(\theta, \phi, t) = \sum_{lm} \int d\omega \, a_{lm}(\omega) Y_l^m(\theta, \phi) e^{i\omega t}$$

$$a_{lm}(\omega) = \frac{1}{2\pi} \int d\Omega dt \; v(\theta, \phi, t) Y_l^{m*}(\theta, \phi) e^{-i\omega t}$$

The k–ω diagram

- The power spectrum

$$p_l(\omega) = \frac{1}{2l+1} \sum_m |a_{lm}(\omega)|^2$$



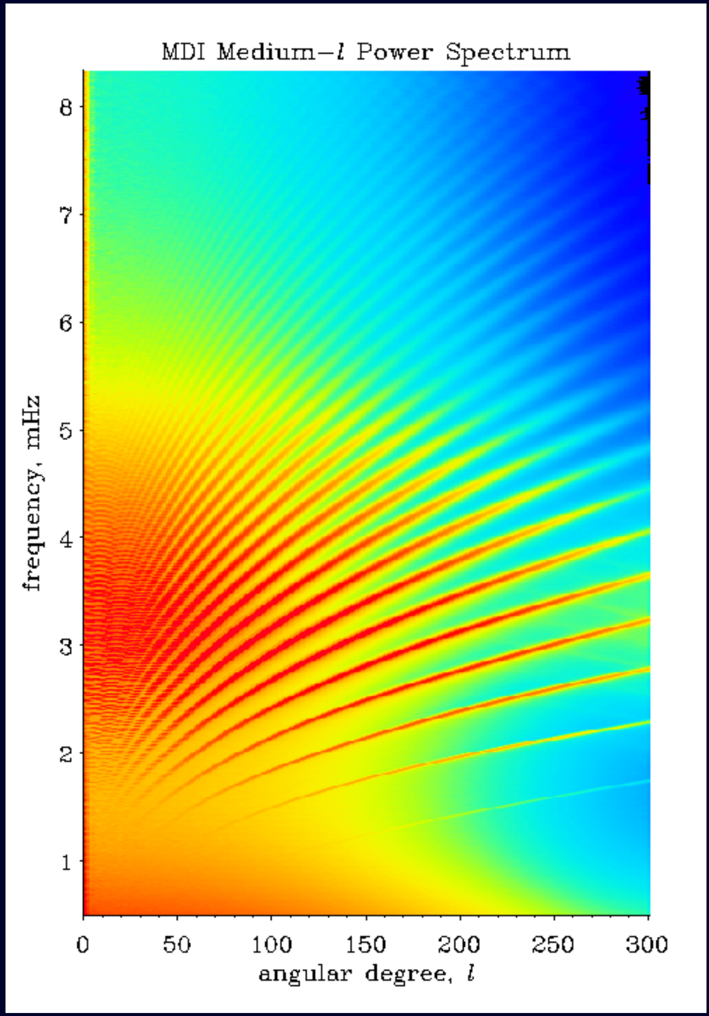
The *k*–ω diagram

- The power spectum

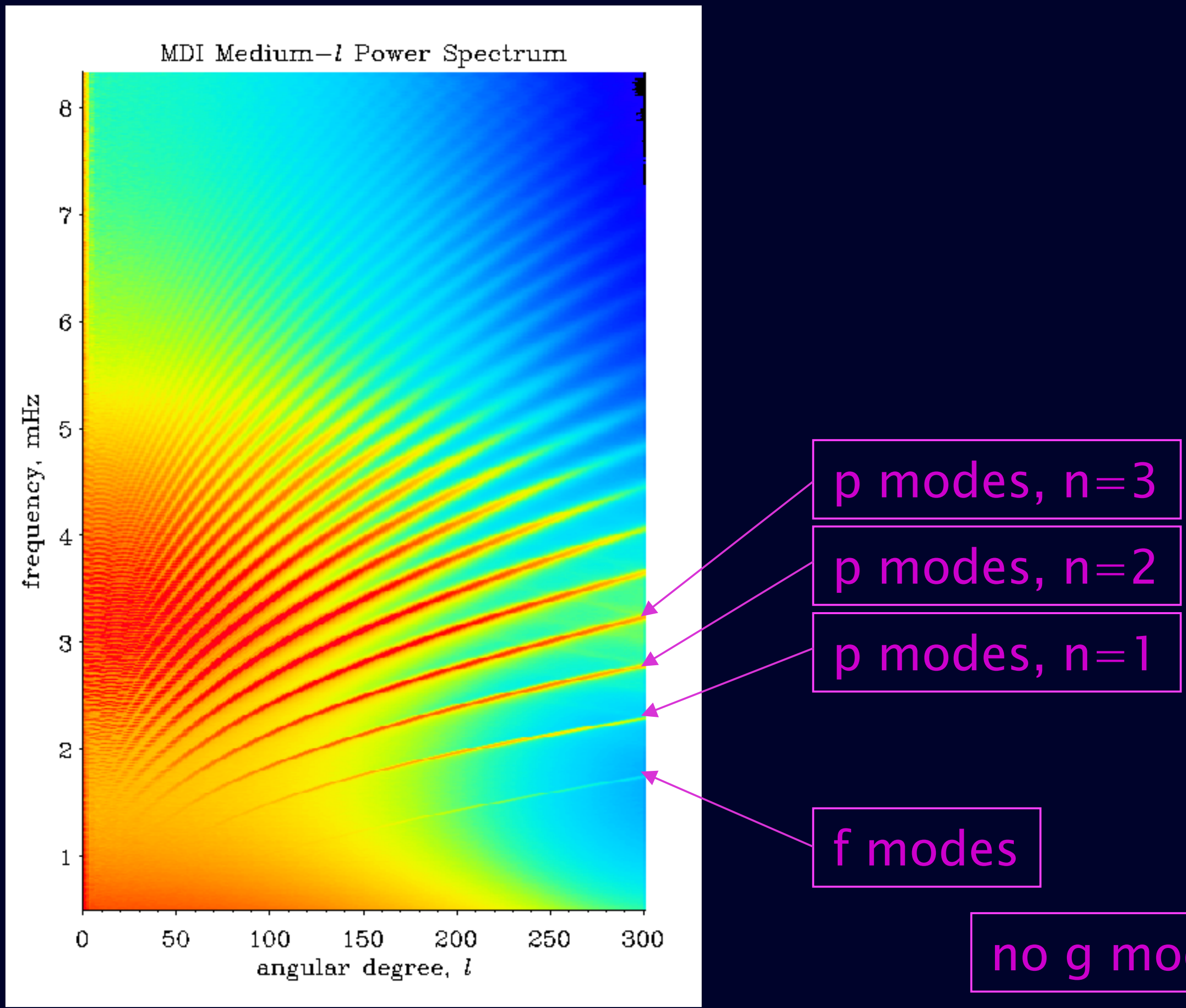
$$p_l(\omega) = \frac{1}{2l+1}\sum_m |a_{lm}(\omega)|^2$$

- The characteristic 'ridge' structure
  - A full explanation would be too lengthy, but it is a signature of acoustic eigenoscillations

p-mode oscillations



The *k*–ω diagram



no g modes yet

Helioseismology

- Solar eigenoscillation frequencies reflect interior structure of the sun

Solar structure → Eigenfrequencies

- In helioseismology, we try to reverse the path

Solar structure ← Eigenfrequencies

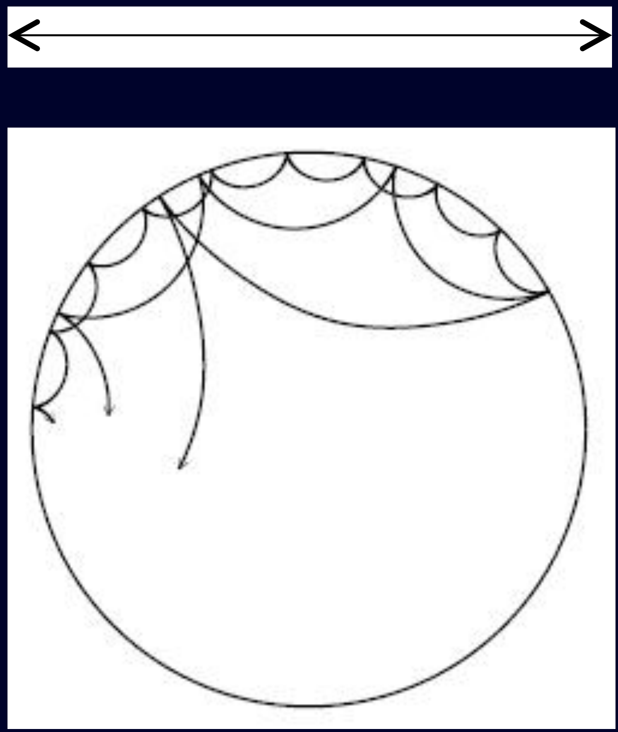
- The principle is the same for asteroseismology

What can helioseismology infer?

- A brief answer: whatever is determining the eigenfrequencies has a chance
- What determines the eigenfrequencies?
- That is to say, what kind of force is working on plasma that constitutes the sun?
  - Gas pressure
  - Gravity
    - Here we are neglecting rotation and magnetic fields

Ray theory

- At the high-frequency ('asymptotic') limit the propagation of sound wave in the sun can be well represented by a ray
- A ray path in the sun is not straight because of the variation in soundspeed



Fluid dynamical equation

- A more precise treatment requires perturbing fluid dynamic equations

$$\omega^2 \rho \vec{\xi} = -\nabla(\rho c^2 \nabla \cdot \vec{\xi}) - \nabla(\nabla P \cdot \vec{\xi}) + \frac{\nabla P}{\rho} \nabla \cdot (\rho \vec{\xi})$$
$$+ \rho \nabla \left[G \int \frac{\nabla' \cdot \left\{ \rho(\vec{r}\,') \vec{\xi}(\vec{r}\,') \right\}}{|\vec{r} - \vec{r}\,'|} dV' \right]$$

$\vec{\xi}(\vec{r})$: displacement vector

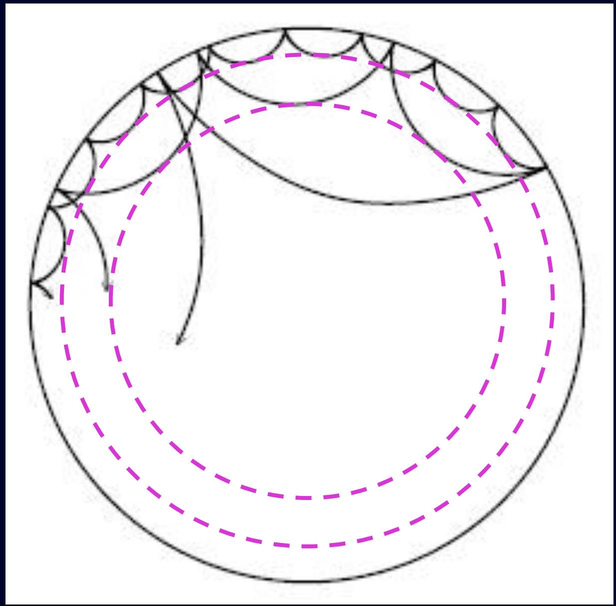
the fluid element at position $\vec{r}$

is now in position $\vec{r} + \vec{\xi}(\vec{r})$

the factor $e^{i\omega t}$ taken out

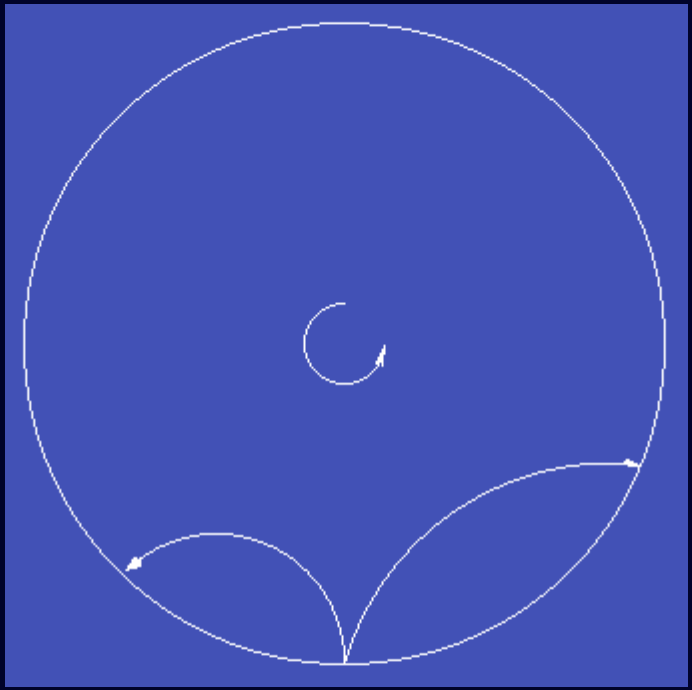
Ray theory

- Modes with smaller ℓ values penetrate deeper
- Different modes 'samples' different parts of the sun
- This is one reason why helioseismology works



Rotation of the sun

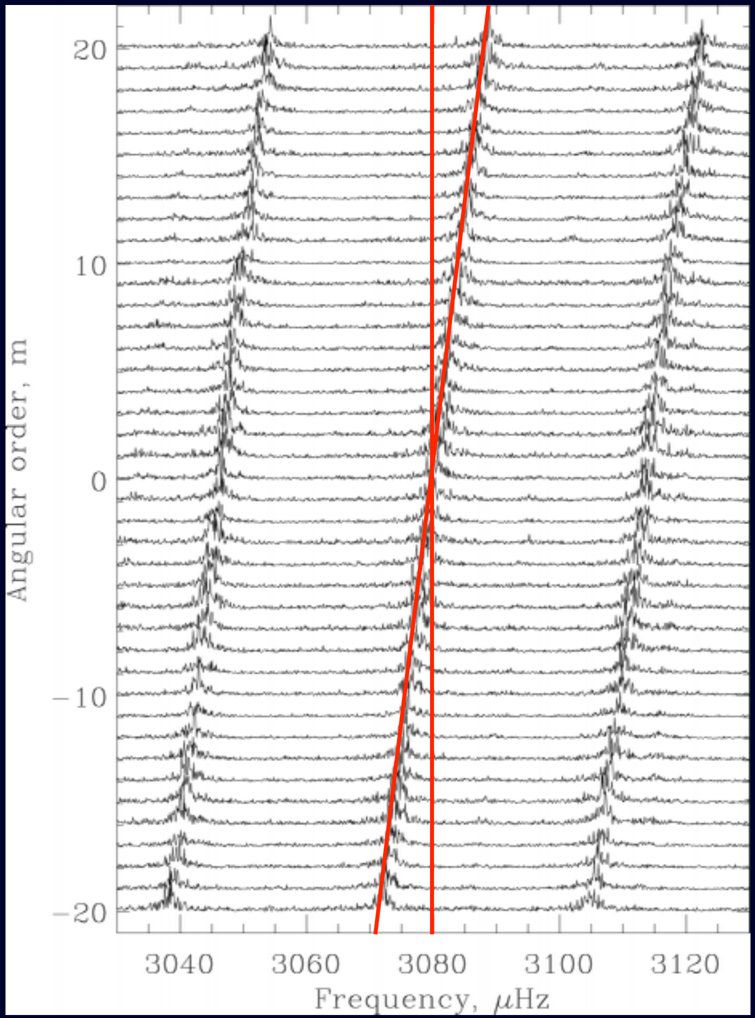
- Rotation of the sun affects the wave propagation
  - primarily by advection
  - also by Coriolis force



Rotational splitting

- As a result, the solar eigenfrequencies are shifted

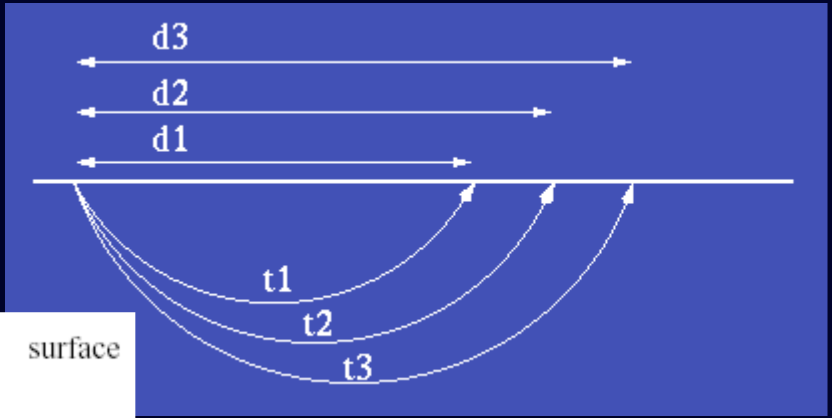
$$|\, a_{lm}(\omega)\,|^2$$

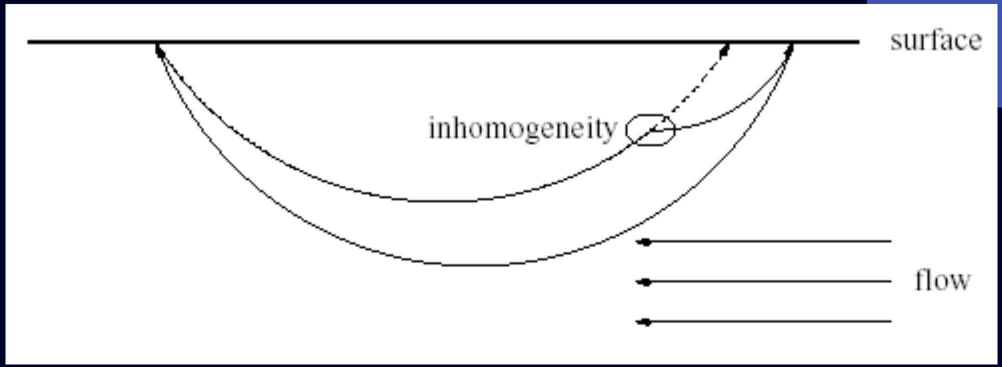


the gradient ∝ rotational frequency

Local helioseismology

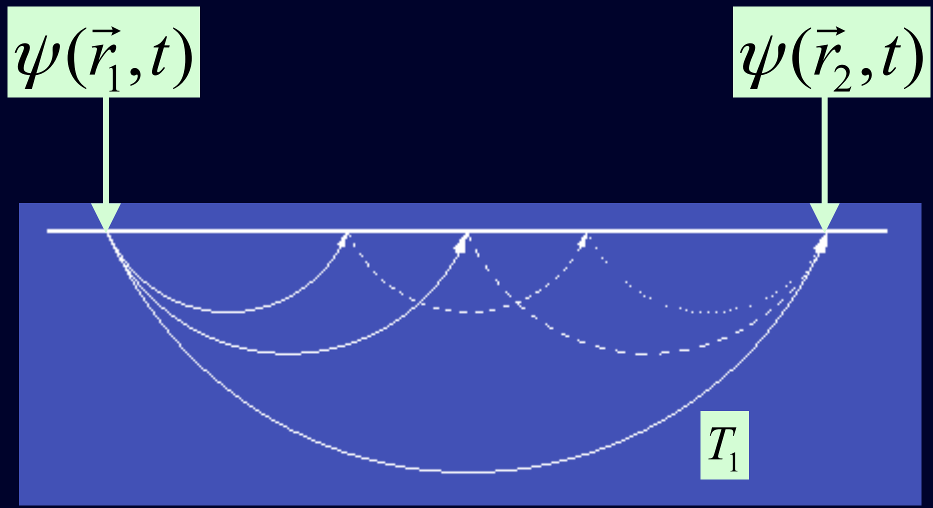
- Forget the global modes
- Direct measurement of subsurface propagation of waves





Time-distance method

- Cross-correlation function

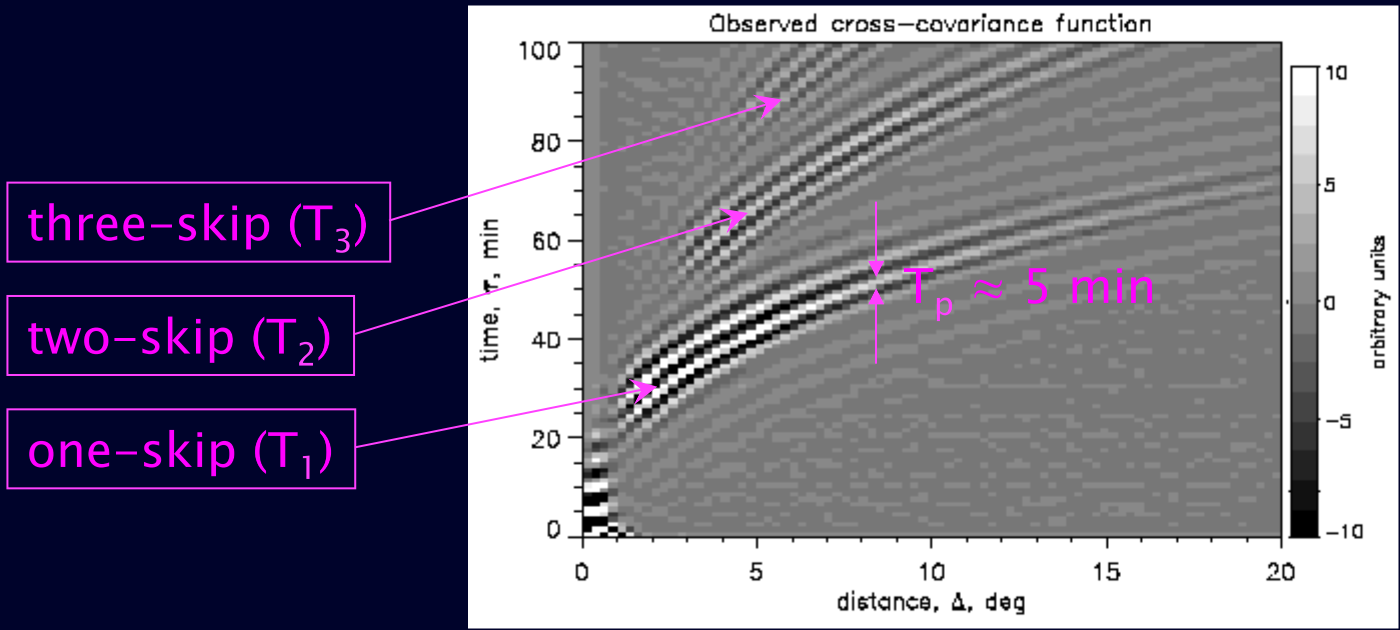


$$C(\vec{r}_1,\vec{r}_2,\tau)=\int\psi^*(\vec{r}_1,t)\psi(\vec{r}_2,t+\tau)dt$$

C is large around $\tau\approx T_1$

Time-distance method

- A solar time-distance diagram



Cross-correlation function

$$C(\Delta,\tau) = \int_{|\vec{r}_1-\vec{r}_2|=\Delta} \psi^*(\vec{r}_1,t)\psi(\vec{r}_2,t+\tau)d\vec{r}_1 d\vec{r}_2 dt$$

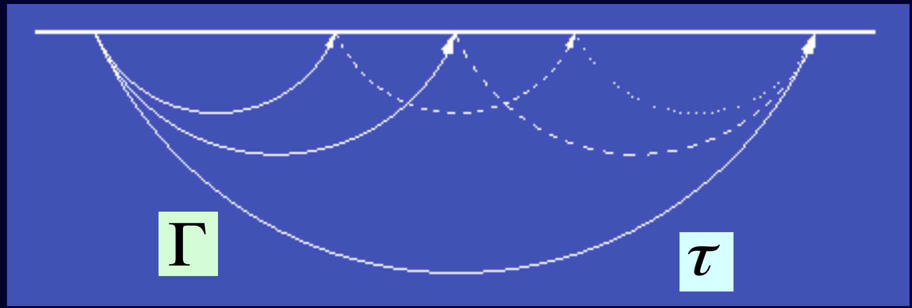
Travel-time perturbation

- Using ray approximation

$$\tau \approx \int_\Gamma \frac{dl}{c} = \int_\Gamma \frac{\vec{k}\cdot d\vec{l}}{\omega}$$

- Soundspeed perturbation δc and flow velocity $\vec{v}$ lead to

$$\delta\tau \approx \frac{1}{\omega}\int_\Gamma \delta\vec{k}\cdot d\vec{l} \approx -\int_\Gamma \frac{\delta c}{c^2}dl - \int_\Gamma \frac{\vec{v}\cdot d\vec{l}}{c^2}$$



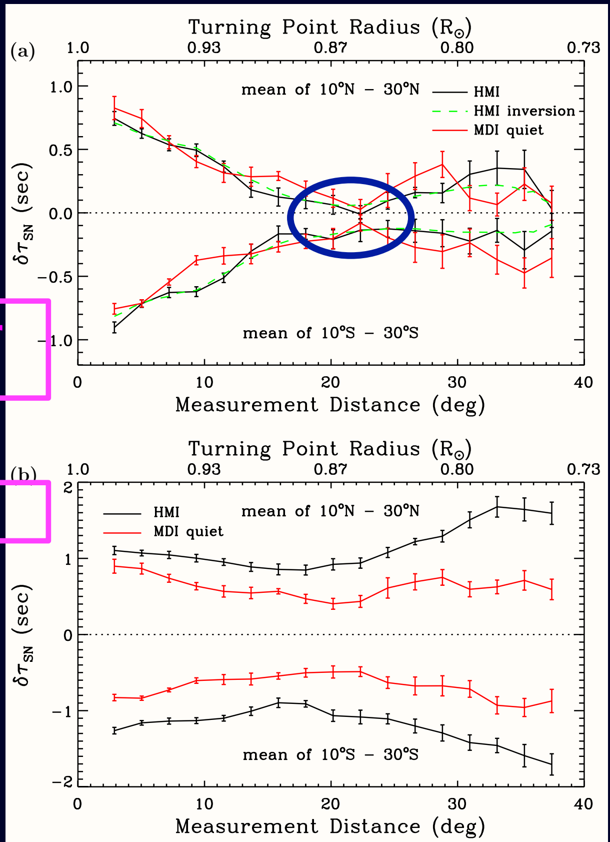
Some more recent results

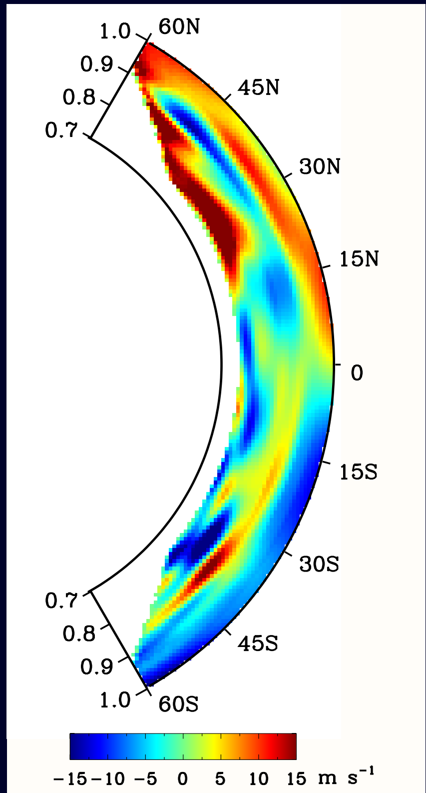
Double-cell meridional flow?

- Zhao et al. (2013)

Corrected for CLV

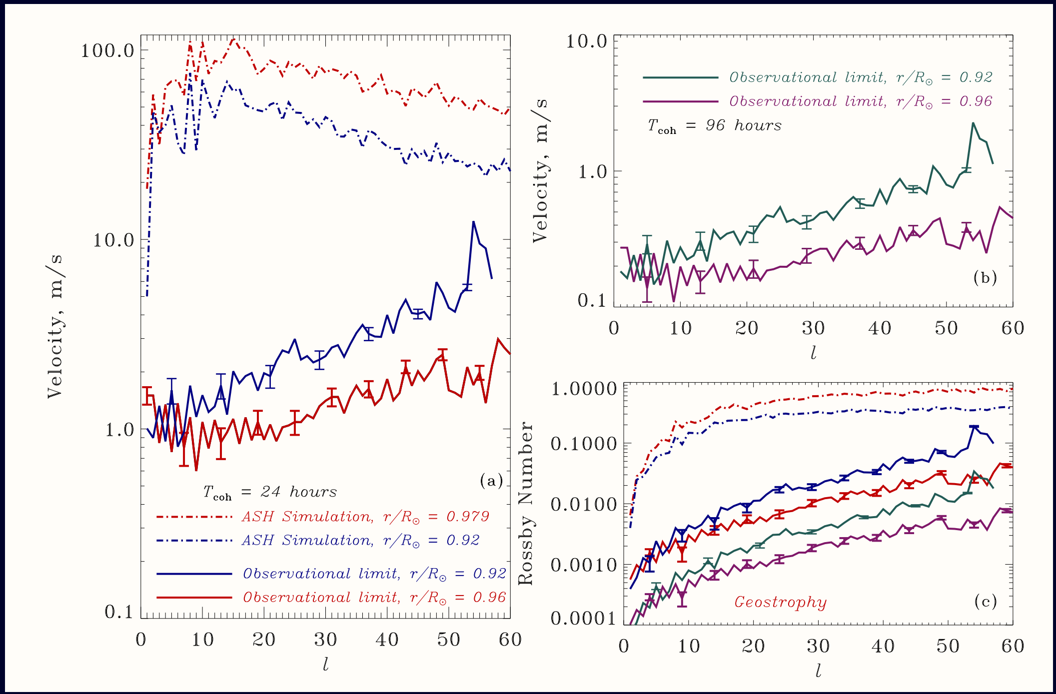
Uncorrected





Convective velocity

- Hanasoge, Duvall & Sreenivasan (2012)
  - Upper limits derived from non detections
  - Convection not so fast?

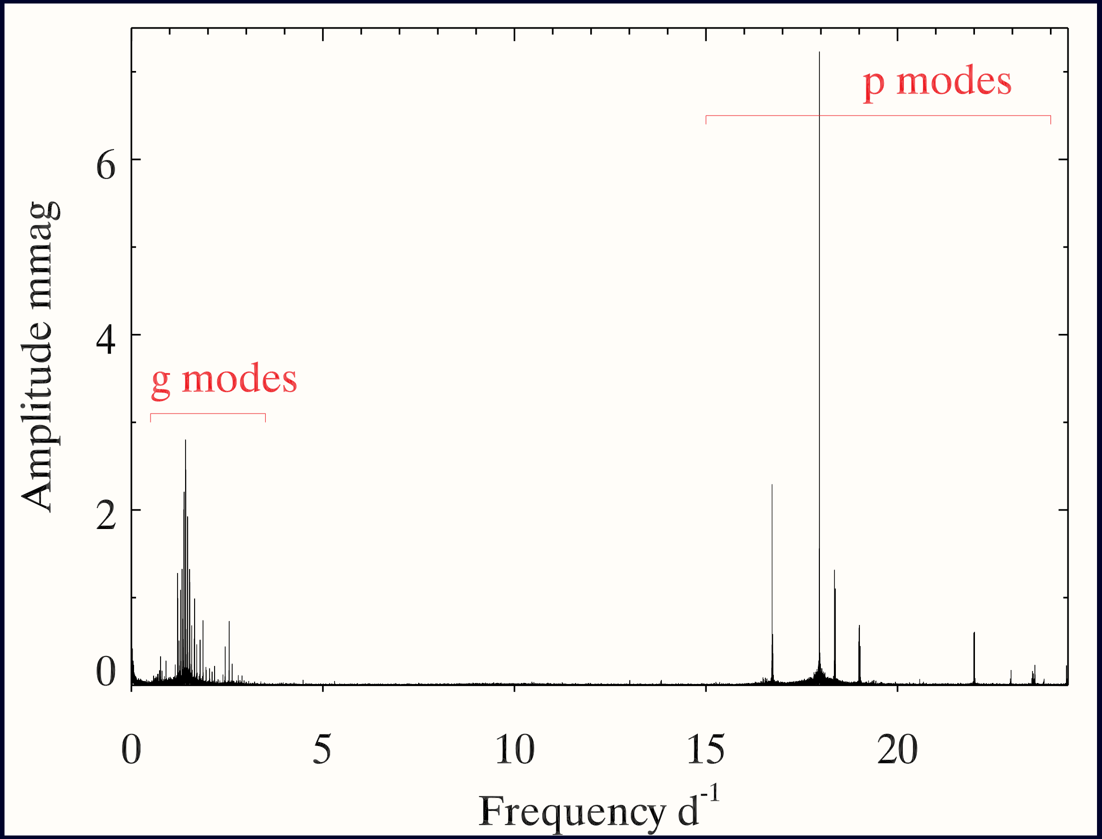


KIC11145123

- A late A star
- Kepler magnitude $K_p = 13$
- Huber et al. (2014)
  - Effective temperature: $T_{eff} = 8050 \pm 200$ K
  - Surface gravity: $\log g = 4.0 \pm 0.2$ (g in cgs)

Oscillations of KIC11145123

- Kepler quarters 0–16, long cadence, 1340-day long



Amplitude spectrum

Oscillations of KIC11145123

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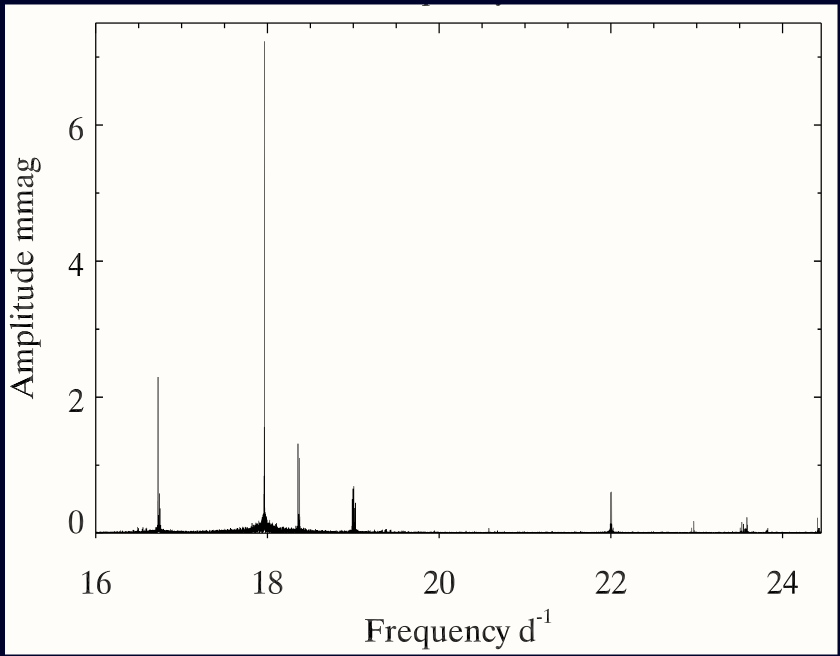

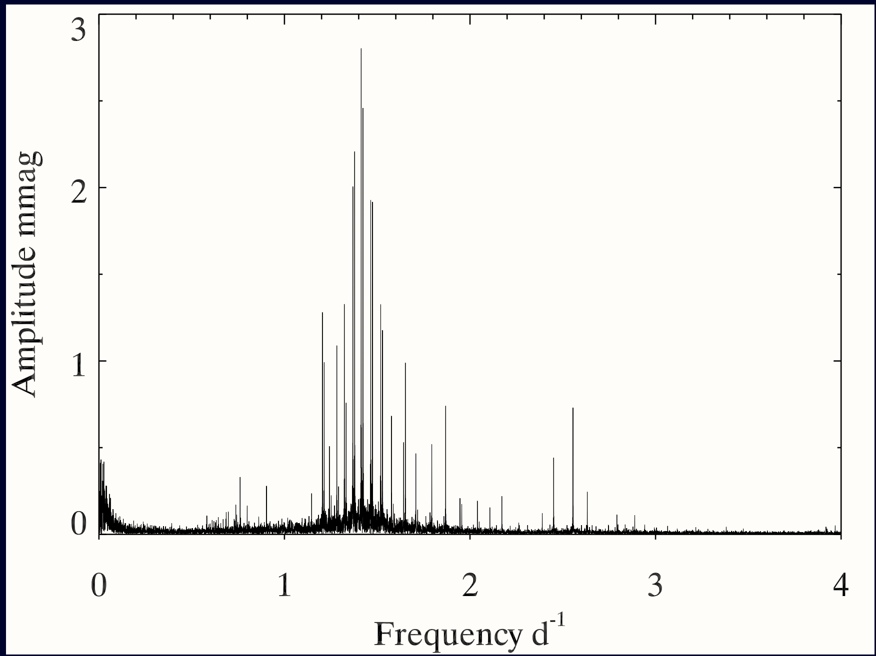

P-mode range

G-mode range

Modelling KIC 11145123

- The best model
  - $M = 1.46 M_{\odot}$
  - Has a convective core (r～0.05R)
  - Z=0.01, Y=0.36
    - Helium abandunce high
    - Too faint and too cool for the KIC parameters

Rotational shift of frequencies

l	n	splt(c/d)	dsplt(c/d)
1	-31	0.0047449	0.0000026
1	-30	0.0047942	0.0000111
1	-29	0.0047512	0.0000071
1	-28	0.0047692	0.0000034
1	-27	0.0047697	0.0000014
1	-26	0.0047566	0.0000011
1	-25	0.0047815	0.0000015
1	-24	0.0047534	0.0000023
1	-23	0.0047865	0.0000112
1	-22	0.0047847	0.0000044
1	-21	0.0048026	0.0000156
1	-20	0.0047939	0.0000181
1	-19	0.0047500	0.0000252
1	-18	0.0047837	0.0000165
1	-17	0.0047745	0.0000296
1	3	0.0101560	0.0000025
1	4	0.0085460	0.0000049
1	5	0.0102570	0.0000599

$$\delta\omega_{nlm} = m(1 - C_{nl}) \int K_{nl}(r)\Omega(r)\,dr$$

Nearly a rigid rotator

- The g-mode splittings show very small scattering
  - $\Delta f_g = 0.0047562 \pm 0.0000023\ \mathrm{d}^{-1}$ (average)
  - Implies a rigid rate of about $0.0095\ \mathrm{d}^{-1}$ (in rotational frequency)
    - $C_{nl} \rightarrow 1/2$ for dipole g modes
- The p-mode shifts are more or less consistent with this rate too
    - $C_{nl} \rightarrow 0$ for p modes
- However…

Rotational shift of frequencies

l	n	splt(c/d)	dsplt(c/d)
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1	5	0.0102570	0.0000599

Core vs envelope

- The envelope seems to be rotating slightly faster since…
  - $\Delta f_g = 0.0047562 \pm 0.0000023\ \mathrm{d}^{-1}$ (average)
  - $\Delta f_p = 1.0101560 \pm 0.0000025\ \mathrm{d}^{-1}$ ($l=1$, $n=3$)
  - $\Delta f_p - 2\Delta f_g > 0$
- Note that
  - $\Omega_p / 2\pi > \Delta f_p$ (lower bound)
  - $\Omega_g / 2\pi < 2\Delta f_g$ (upper bound)

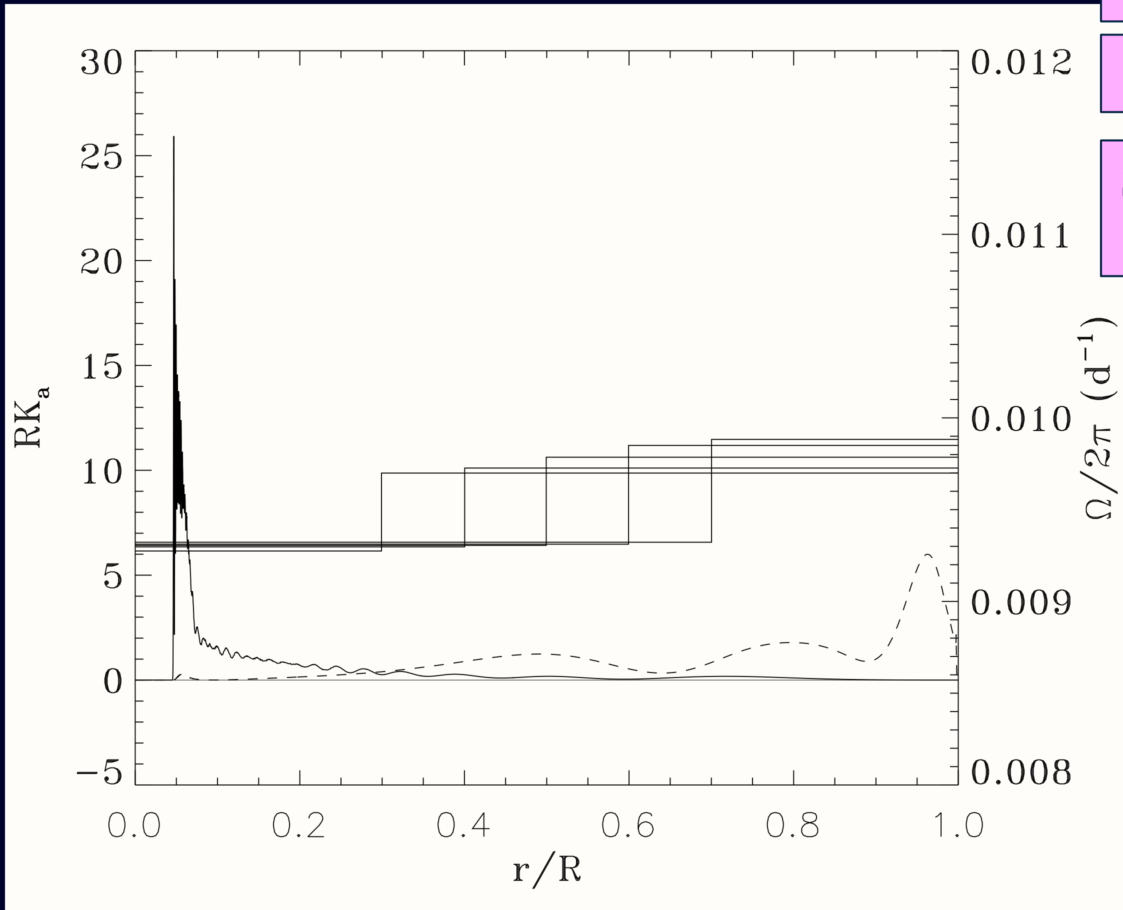
Two-zone modelling

- Fitting the following form

$$\Omega(r) = \begin{cases} \Omega_1 & (0 \le r \le r_b) \\ \Omega_2 & (r_b \le r \le R) \end{cases}$$

- …not even to individual splittings, but to the p- and g-averaged splittings

Two-zone modelling



Solid: g-mode kernel

Dotted: p-mode kernel

Step functions: two-zone models

Good separation between regions sampled by two kernels

KIC 11145123 results

- A terminal-age main-sequence A star KIC11145123 exhibit both p-mode oscillations and g-mode oscillations
- This permits us to examine the core rotation and the envelope rotation separately
  - The star is almost a rigid rotator
  - The envelope however is rotating slightly faster ‘on average’
- There are implications on angular momentum transport mechanism --- waves?
  - Strong mechanism inferred from red-giant results too

Summary

- Helioseismology is about measuring physical quantities *inside* the sun, based on wave/oscillation
  - Global mode inversions have revealed the internal structure as well as the internal differential rotation
  - Local helioseismology, still an immature discipline, promises to tell us more about inhomogeneous static and dynamic structure of the sun
- …and then to asteroseismology!