A brief introduction to helio- and asteroseismology

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Helio- and asteroseismology

- A quick overview
  - What they are
  - Why they are interesting
  - What we have learned
- How do they work?
  - More physical and mathematical details
- Some more recent results

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A quick overview
Helio- and asteroseismology

- Investigation of the solar and stellar interiors based on their oscillations
  - The Sun and stars are not transparent to electromagnetic waves
  - They are transparent to ‘waves’
  - Seismic approaches are the only way to study inner work of the Sun and stars
Helio- and asteroseismology

- Oscillations?
  - Variable stars are known from ancient times and many of them are pulsating variables.
  - Since 1962, the Sun is also known to be a pulsating variable.

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Global vs. Local

- Two main flavours of helioseismology
  - Global helioseismology
    - based on global eigenfrequencies
    - for ‘global’ or highly symmetric structures
  - Local helioseismology
    - based on (for example) local travel times
    - for localized measurements

- Asteroseismology can only be ‘global’
What do we want to study?

- Is the standard solar model correct?
  - Do we understand solar/stellar evolution processes?

- Ingredients of solar dynamo
  - Differential rotation, meridional flow
  - Convection

- How the internal processes are connected to observable surface phenomena
  - Subsurface ‘weather’
  - Flux emergence processes

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Soundspeed inversion

- The depth of the convection zone is about 20 Mm, not 15 Mm
- The first major result from helioseismology

\[ c^2 \propto \frac{T}{\mu} \]

The base of the convection zone

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Soundspeed inversion

- This modern model agrees with the ‘observation’ within a half per cent accuracy.

What’s this bump?

Rather uncertain

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Solar differential rotation

- Based on measurement of rotational shifts of eigenfrequencies

- Surface shear layer
- Non-columnar rotation
- ‘Tachocline’
- Rigid rotation?

Previous dynamo calculations all rejected

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Tachocline

- A steep gradient in the rotation rate

Dynamo action?

Shear-induced mixing counteracting gravitational settling of Helium?

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Structure around a sunspot

- From time-distance method

A very well-known result but nobody is sure if this is correct

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Double–cell meridional flow?

- Zhao et al. (2013)

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Supergranulation

- ‘Divergent’ flow signatures

Travel-time difference (inward/outward) maps (15-Mm scale)

Equatorial (Sekii et al. 2007)

Polar region (Nagashima et al. 2011)

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Supergranulation

- Helioseismic analyses are finding out that supergranulation is a fairly ‘shallow’ phenomenon
  - Duvall 1998 (MDI), Sekii et al 2007 (Hinode/SOT)
  - ~10Mm deep, give or take
- Polar supergranules are smaller and deeper than their low-latitude counterparts? (Nagashima 2010)
Emerging flux detections

- Can we detect magnetic fluxes before their emergence?

A travel-time perturbation map

Ilonidis, Zhao & Kosovichev (2011)
Far-side imaging

☐ The other side of the sun
How do they work?
Dopplergram movies

- From SOHO/MDI
Dopplergram

- Dopplergram obtained by SOHO/MDI

-2km/s < v < 2km/s

Dominated by solar differential rotation
Dopplergram

- By subtracting 45-min average we can filter out rotation and supergranulation.
FSH decomposition

- Any scalar function on sphere can be expanded in spherical harmonics

\[ f(\theta, \phi) = \sum_{lm} f_{lm} Y_l^m(\theta, \phi) \]

\[ Y_l^m(\theta, \phi) = P_l^m(\cos \theta)e^{im\phi} \]

- \( l \): degree
- \( m \): azimuthal order

\[ f(\theta, \phi) \text{ symmetric} \]
\[ \Rightarrow f_{lm} \text{ indep't of } m \]
FSH decomposition

- For simplicity, we assume we observe the radial velocity (rather than the line-of-sight velocity)
- In spatial domain, the velocity field can be expanded in spherical harmonics

\[
\nu(\theta, \phi, t) = \sum_{lm} A_{lm}(t)Y_{lm}^m(\theta, \phi)
\]

- \(\nu(\theta, \phi, t)\): radial velocity field
- \(Y_{lm}^m(\theta, \phi)\): spherical harmonic function with degree \(l\) and azimuthal order \(m\)
FSH decomposition

- In time domain, Fourier decomposition comes in handy.

\[ A_{lm}(t) = \int a_{lm}(\omega)e^{i\omega t} \, d\omega \]

- Then we have Fourier–Spherical–Harmonic decomposition of the velocity field.

\[ v(\theta, \phi, t) = \sum_{lm} \int d\omega \, a_{lm}(\omega)Y_l^m(\theta, \phi)e^{i\omega t} \]

\[ a_{lm}(\omega) = \frac{1}{2\pi} \int d\Omega dt \, v(\theta, \phi, t)Y_l^m(\theta, \phi)e^{-i\omega t} \]
The $k-\omega$ diagram

- The power spectrum

$$p_l(\omega) = \frac{1}{2l + 1} \sum_{m} |a_{lm}(\omega)|^2$$
The $k$–$\omega$ diagram

- The power spectrum
  \[ p_l(\omega) = \frac{1}{2l+1} \sum_m |a_{lm}(\omega)|^2 \]

- The characteristic ‘ridge’ structure
  - A full explanation would be too lengthy, but it is a signature of acoustic eigenoscillations
  - p-mode oscillations
The $k$–$\omega$ diagram

- p modes, $n=1$
- p modes, $n=2$
- p modes, $n=3$
- f modes
- no g modes yet
Helioseismology

- Solar eigenoscillation frequencies reflect interior structure of the sun

- In helioseismology, we try to reverse the path

- The principle is the same for asteroseismology
What can helioseismology infer?

- A brief answer: whatever is determining the eigenfrequencies has a chance
- What determines the eigenfrequencies?
- That is to say, what kind of force is working on plasma that constitutes the sun?
  - Gas pressure
  - Gravity
    - Here we are neglecting rotation and magnetic fields
Ray theory

- At the high-frequency (‘asymptotic’) limit the propagation of sound wave in the sun can be well represented by a ray.
- A ray path in the sun is not straight because of the variation in soundspeed.
Fluid dynamical equation

- A more precise treatment requires perturbing fluid dynamic equations

\[
\omega^2 \rho \ddot{\xi} = -\nabla(\rho c^2 \nabla \cdot \ddot{\xi}) - \nabla(\nabla P \cdot \ddot{\xi}) + \frac{\nabla P}{\rho} \nabla \cdot (\rho \ddot{\xi}) \\
+ \rho \nabla \left[ G \int \frac{\nabla' \cdot \left\{ \rho(\vec{r}') \ddot{\xi} (\vec{r}') \right\} dV'}{|\vec{r} - \vec{r}'|} \right]
\]

\(\ddot{\xi}(\vec{r})\): displacement vector
the fluid element at position \(\vec{r}\)
the factor \(e^{i\omega t}\) taken out
is now in position \(\vec{r} + \ddot{\xi}(\vec{r})\)
Ray theory

- Modes with smaller $\ell$ values penetrate deeper
- Different modes ‘samples’ different parts of the sun
- This is one reason why helioseismology works
Rotation of the sun

- Rotation of the sun affects the wave propagation
  - primarily by advection
  - also by Coriolis force
Rotational splitting

As a result, the solar eigenfrequencies are shifted

\[ |a_{lm}(\omega)|^2 \]

the gradient \( \propto \) rotational frequency
Local helioseismology

- Forget the global modes
- Direct measurement of subsurface propagation of waves
Time–distance method

- Cross–correlation function

\[ C(\vec{r}_1, \vec{r}_2, \tau) = \int \psi^*(\vec{r}_1, t) \psi(\vec{r}_2, t + \tau) dt \]

\[ C \text{ is large around } \tau \approx T_1 \]
Time–distance method

- A solar time–distance diagram

Cross-correlation function

\[ C(\Delta, \tau) = \int_{|\vec{r}_1 - \vec{r}_2| = \Delta} \psi^*(\vec{r}_1, t)\psi(\vec{r}_2, t + \tau) \, d\vec{r}_1 \, d\vec{r}_2 \, dt \]

three-skip \( (T_3) \)
two-skip \( (T_2) \)
one-skip \( (T_1) \)

\( T_p \approx 5 \text{ min} \)
Travel-time perturbation

- Using ray approximation

- Soundspeed perturbation and flow velocity lead to

\[ \tau \approx \int_{\Gamma} \frac{dl}{c} = \int_{\Gamma} \frac{k \cdot dl}{\omega} \]

\[ \delta \tau \approx \frac{1}{\omega} \int_{\Gamma} \delta k \cdot d\mathbf{l} \approx -\int_{\Gamma} \frac{\delta c}{c^2} dl - \int_{\Gamma} \frac{\dot{\mathbf{v}} \cdot d\mathbf{l}}{c^2} \]
Some more recent results
Double-cell meridional flow?

☐ Zhao et al. (2013)
Convective velocity

- Hanasoge, Duvall & Sreenivasan (2012)
  - Upper limits derived from non detections
  - Convection not so fast?

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KIC11145123
- A late A star
- Kepler magnitude $K_p = 13$
- Huber et al. (2014)
  - Effective temperature: $T_{\text{eff}} = 8050 \pm 200$ K
  - Surface gravity: $\log g = 4.0 \pm 0.2$ (g in cgs)
Oscillations of KIC11145123

- Kepler quarters 0–16, long cadence, 1340–day long

![Amplitude spectrum](https://archive.stsci.edu/kepler/data/)
Oscillations of KIC11145123

- Kepler quarters 0–16, long cadence, 1340–day long

Figure 1. Top panel: An amplitude spectrum for the Q0–16 Kepler Mission photometric data. The mid- and bottom panels show expanded looks in the p-mode and g-mode frequency regions, respectively. Modes with positive (negative) frequency differences from the solar fundamental frequency (17 μHz) are g (p) modes. The indices \( n/2 \) and \( \ell \) indicate the number of surface nodes, \( m/2 \) is the radial degree and the azimuthal order, respectively. These indices, along with \( \nu \) and \( \delta \), specify the structure of the eigenfunction (e.g., the radial degree and the azimuthal order). The frequencies \( \nu \) and \( \delta \) are related by \( \nu = \delta + \nu_{\text{solar}} \), where \( \nu_{\text{solar}} \) is the solar fundamental frequency (17 μHz).

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Modelling KIC 11145123

- The best model
  - $M=1.46M_\odot$
  - Has a convective core ($r \sim 0.05R$)
  - $Z=0.01$, $Y=0.36$
    - Helium abundance high
    - Too faint and too cool for the KIC parameters
Rotational shift of frequencies

\[ \delta \omega_{nlm} = m(1 - C_{nl}) \int K_{nl}(r) \Omega(r) \, dr \]

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Nearly a rigid rotator

- The g-mode splittings show very small scattering
  - $\Delta f_g = 0.0047562 \pm 0.0000023 \, \text{d}^{-1}$ (average)
  - Implies a rigid rate of about 0.0095 d$^{-1}$ (in rotational frequency)
    - $C_{nl} \rightarrow 1/2$ for dipole g modes

- The p-mode shifts are more or less consistent with this rate too
  - $C_{nl} \rightarrow 0$ for p modes

- However…
Rotational shift of frequencies

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Core vs envelope

- The envelope seems to be rotating slightly faster since...
  - $\Delta f_g = 0.0047562 \pm 0.0000023 \text{ d}^{-1}$ (average)
  - $\Delta f_p = 1.0101560 \pm 0.0000025 \text{ d}^{-1}$ (l=1, n=3)
  - $\Delta f_p - 2\Delta f_g > 0$

- Note that
  - $\Omega_p/2\pi > \Delta f_p$ (lower bound)
  - $\Omega_g/2\pi < 2\Delta f_g$ (upper bound)
Two-zone modelling

- Fitting the following form

\[ \Omega(r) = \begin{cases} 
\Omega_1 & (0 \leq r \leq r_b) \\
\Omega_2 & (r_b \leq r \leq R) 
\end{cases} \]

- ...not even to individual splittings, but to the p- and g-averaged splittings
Two–zone modelling

Solid: g-mode kernel
Dotted: p-mode kernel
Step functions: two-zone models

Good separation between regions sampled by two kernels

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KIC 11145123 results

- A terminal-age main-sequence A star KIC11145123 exhibit both p-mode oscillations and g-mode oscillations.
- This permits us to examine the core rotation and the envelope rotation separately.
  - The star is almost a rigid rotator.
  - The envelope however is rotating slightly faster ‘on average’.
- There are implications on angular momentum transport mechanism --- waves?
  - Strong mechanism inferred from red-giant results too.

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Summary

- Helioseismology is about measuring physical quantities *inside* the sun, based on wave/oscillation
  - Global mode inversions have revealed the internal structure as well as the internal differential rotation
  - Local helioseismology, still an immature discipline, promises to tell us more about inhomogeneous static and dynamic structure of the sun

- ...and then to asteroseismology!